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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

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ON THE ALBEDO OF THE PLANETS AND THEIR SATELLITES

By HENRY NORRIS RUSSELL

In a preceding paper, the photometric observations of the sun, moon, and planets were reviewed, and their stellar magnitudes derived. In the present communication, values of the albedo of the various bodies are deduced from these data, and the material bearing on the albedo of the earth is considered.

I. THE DEFINITIONS OF ALBEDO, AND THE THEORY OF ITS DETERMINATION

According to Lambert's original definition[1] of the albedo of a diffusely reflecting surface, it is the ratio of the amount of light diffusely reflected in all directions by an element of this surface to the incident amount of light which falls on this element. With the law of diffuse reflection proposed by Lambert, this ratio is the same for all angles of incidence; but Seeliger[2] has shown that for other laws of reflection, whether derived from theoretical assumptions or from observation, this is not generally the case. Following his notation, suppose that an element $d\sigma$ of the surface is placed in a beam of parallel light of intensity L (such that a unit of area

[1] See Müller, *Photometrie der Gestirne* (Leipzig, 1897), pp. 52–55.

[2] *Abhandlungen der K. bayer. Akad. d. Wissenschaften*, **16**, 430, 1888.

under normal incidence would receive the quantity of light L). Under any angle of incidence i the element will receive the amount $L \cos i \, d\sigma$. The intensity of the diffusely reflected light, at unit distance, will in general depend both on the angle of emanation ϵ and on the azimuth of the plane of reflection. The mean intensity for all azimuths, and a given value of ϵ, will be proportional to L and $d\sigma$, and may be expressed in the form

$$\gamma L \, d\sigma f(i, \epsilon),$$

where γ is a constant. Integrating over a hemisphere of unit radius described about the element, to find the whole amount of reflected light, and dividing by the amount of the incident light, the albedo, according to Lambert's definition, is given by the equation

$$\mu' = 2\pi\gamma \int_0^{\frac{\pi}{2}} \frac{f(i, \epsilon)}{\cos i} \sin \epsilon \, d\epsilon. \qquad (1)$$

This will be a function of i unless the law of reflection is such that

$$f(i, \epsilon) = \cos i \, f(\epsilon).$$

If then we are to speak of *the* albedo of the surface, we must choose in some more or less arbitrary way,[1] either the value of μ' for some particular angle of incidence, as a mean value for all such angles. Seeliger decides to weight the different values of μ' according to the probability of their occurrence, which is proportional to $\sin i$, and so obtains, as the definition of the albedo,

$$\mu = \int_0^{\frac{\pi}{2}} \mu' \sin i \, di = 2\pi\gamma \int_0^{\frac{\pi}{2}} \tan i \, di \int_0^{\frac{\pi}{2}} f(i, \epsilon) \sin \epsilon \, d\epsilon, \qquad (2)$$

which may be expressed verbally as follows: "An element of surface is successively illuminated by beams of parallel light coming at random from all directions. The *mean of the ratios* of the diffusely reflected light to the incident light is the albedo."

Other definitions might equally well be given; for example: "An element of surface is exposed to rays of parallel light coming from all directions at random, or from a uniformly bright sky. The

[1] "Einige Willkür ist natürlich hierbei nicht zu umgehen."—Seeliger, *loc. cit.*

ratio of the whole amount of light diffusely reflected to the *whole amount* incident on the element is its albedo." In this case, if the intensity of the light coming from unit solid angle of the sky is L, the amount of light which reaches the element under angles of incidence between i and $i+di$ will be

$$2\pi L \cos i \sin i \, di \, d\sigma ,$$

of which the fraction μ' is reflected. Integrating over the hemisphere, it is found that the whole amount of incident light is $\pi L d\sigma$, and that the albedo on this definition is given by the equation

$$A = 2\int_0^{\frac{\pi}{2}} \mu' \sin i \cos i \, di = 4\pi\gamma \int_0^{\frac{\pi}{2}} \sin i \, di \int_0^{\frac{\pi}{2}} f(i, \epsilon) \sin \epsilon \, d\epsilon . \qquad (3)$$

Still another definition, and from a more definitely astronomical viewpoint, was proposed long ago by Bond,[1] as follows: "Let a sphere S be exposed to parallel light. Then its albedo A is the ratio of the whole amount of light reflected from S to the whole amount incident on it." This definition leads again to the equation (3). If r is the radius of the sphere, and L the intensity of the incident light, the whole amount incident on the sphere is $\pi r^2 L$. The zone of the sphere upon which the angle of incidence is between i and $i+di$, as seen from the direction of the light, has the projected area $2\pi r^2 \cos i \sin i \, di$, and reflects the fraction μ' of the incident light. Hence it is easily found that

$$A = 2\int_0^{\frac{\pi}{2}} \mu' \sin i \cos i \, di ,$$

as above.

Unless μ' is independent of i, Bond's and Seeliger's definitions will lead to different values for the albedo of the same body, the difference arising because Seeliger's gives much greater weight to the small angles of incidence, in spite of the fact that the amounts of both incident and reflected light under these conditions are small. From the astronomical standpoint, several reasons can be urged

[1] *Proceedings of the American Academy of Arts and Sciences*, N.S., **8**, 232, 1861.

in favor of Bond's definition; for example: (1) It corresponds more closely to the actual circumstances of observation. (2) In dealing with such questions as the effect of solar radiation on the temperature of a planet, where the amount of heat absorbed by its surface is the determining factor, Bond's definition of the albedo will always lead to correct results, while Seeliger's may not. (3) The law of diffuse reflection $f(i, \epsilon)$, and the law connecting μ' with i are unknown for the planetary surfaces, so that computations by Seeliger's formulae must rest on more or less arbitrary assumptions. (4) For a number of bodies, however (the moon, Mercury, Venus, and in part for Mars), the relation between the amounts of light reflected in different directions—that is, at different phase-angles—is known from observation, so that the albedo on Bond's definition can be computed independently of all assumptions. The last point is important. It is well known that the manner in which the brightness varies with the phase-angle is very different for different bodies, and that not one of them agrees with any of the laws derived from theoretical assumptions, so that an empirical law must be derived from the observations in each case.

If the brightness of the planet (reduced to standard distances from the earth and sun) at the full phase be taken as unit, that at any other phase-angle α may be called $\phi(\alpha)$. The whole amount of light reflected by the planet to the celestial sphere will be proportional to

$$\int_0^\pi \phi(\alpha) \sin \alpha \, d\alpha \, .$$

If it shone in all directions with the brightness of the full phase, the emitted light would be 2.0 on the same scale. Now let r be the mean radius of the planet's disk, and R its distance from the sun, and M_0 be the ratio of the apparent brightness of the planet at the full phase, and at distance Δ from the earth, to that of the sun at unit distance. The fraction of the sun's whole radiation which the planet intercepts is $\frac{r^2}{4R^2}$. If it shone in all directions with its full brightness, the whole amount of emitted light would be $M_0\Delta^2$ times

that emitted by the sun. The albedo of the planet, according to Bond's definition, is therefore given by the equation

$$A=\frac{R^2M_0\Delta^2}{r^2}\cdot 2\int_0^\pi \phi(\alpha)\sin\alpha\, d\alpha\,. \tag{4}$$

The albedo here appears as the product of two factors, of which one depends only on the geometrical and photometric relations of the planet as observed at the full phase, and the other entirely upon the law of variation of its brightness with phase. The first factor, which may be called p, may be expressed in a variety of forms. If σ is the angular semi-diameter of the planet as seen from the earth, s that of the sun as seen from the planet, and S that of the sun seen from the earth, then

$$p=\frac{M_0R^2\Delta^2}{r^2}=\frac{M_0\sin^2 S}{\sin^2\sigma\sin^2 s}=\frac{M_0R^2}{\sin^2\sigma}\,, \tag{5}$$

which are well-known formulae.

If σ_1 denotes the semi-diameter of the planet in seconds of arc, and g its stellar magnitude at the full phase, reduced in both cases to unit distance from the earth and sun, and if G is the sun's stellar magnitude at unit distance, it is easily shown that

$$\log_{10}p=\frac{2}{5}(G-g)-2\log_{10}\sin\sigma_1\,.$$

Introducing the value of G (-26.72 on the Harvard scale)[1] and remembering that

$$\log_{10}\sin\sigma_1=\log_{10}\sigma_1-5.3144\,,$$

this becomes

$$\log_{10}p=-\frac{2}{5}(g+0.15)-2\log_{10}\sigma_1\,. \tag{6}$$

The factor p may also be defined as the ratio of the actual brightness of the planet at the full phase to that of a self-luminous body of the same size and position, which radiates as much light from

[1] *Astrophysical Journal*, **43**, 105, 1916.

each unit of its surface as the planet receives from the sun under normal illumination.

If now

$$q=2\int_0^\pi \phi(a)\sin a\, da\,, \qquad A=pq\,. \tag{7}$$

The factor q is different for the various laws of diffuse reflection, and still more so for the planets as they actually are. For the principal theoretical laws it is found that, for a sphere,[1] on Lambert's law:

$$\phi(a)=\frac{1}{\pi}(\sin a+(\pi-a)\cos a),\ \text{and}\ q=\frac{3}{2}\,;$$

on the Lommel-Seeliger law:

$$\phi(a)=1-\sin\frac{a}{2}\tan\frac{a}{2}\log\cot\frac{a}{4}\,,\ q=(1-\log 2)\frac{16}{3}=1.6366$$

and on Euler's law:

$$\phi(a)=\cos^2\frac{a}{2}\,,\ q=2\,.$$

For the Lommel-Seeliger law,

$$f(i,\epsilon)=\frac{\cos i\cos\epsilon}{\cos i+\cos\epsilon}\,,$$

and it is found that

$$\mu'=2\cdot\pi\gamma[1-\cos i\log(1+\sec i)]\,,\quad A=\frac{8}{3}\pi\gamma(1-\log 2)=0.8183\,\pi\gamma\,.$$

The factor in brackets in the first of these expressions ranges from 0.308 when $i=0°$ to unity when $i=90°$. Since μ' can never exceed unity, it follows that $\pi\gamma$ cannot be greater than 0.5, nor A than 0.409. Hence a planet for which p exceeds 0.25 cannot reflect light in strict accordance with the Lommel-Seeliger law.

For those heavenly bodies whose brightness can be observed over a sufficient range of phase-angles, the value of q can be determined by mechanical quadrature.[2] Since in all cases the falling off

[1] For a flat disk, normally illuminated and viewed, the value of q is 1.0 if it reflects according to Lambert's law, and the same as for a sphere on the other two laws.

[2] There is of course a gap in the observed curves near each conjunction with the sun, widest near inferior conjunction. But since the factor $\sin a$ in the integrand is here small, a free-hand extension of the observed curves suffices to obtain reliable values of q.

in brightness with advancing phase is more rapid than is predicted by the simple theories, the values of q are smaller. Müller's observations of Venus (treated in this way) lead to the value 1.194 for q, while the light-curve derived in the previous paper for the moon gives $q=0.694$. For Mercury, Müller gives two empirical formulae. One gives practically the same value as for the moon; while the other, which agrees better with an observation made near full phase during a total solar eclipse, makes $q=0.420$.

It is therefore clear that, in the absence of knowledge of the law of variation with phase, little can be told about the albedo of a planet by observations of the brightness at full phase alone. Indeed, observations made when the phase is decidedly gibbous are actually better for this purpose. If $p(\alpha)$ denotes the value of p computed from the brightness at phase-angle α instead of at the full phase, the albedo is $p\ (\alpha)\ \dfrac{q}{\phi(\alpha)}$. The values of the second factor, for the six different laws of variation with phase which are considered above, are given in Table I.

TABLE I

α	Euler	Lambert	Seeliger	Venus	Moon	Mercury
0°.........	2.00	1.50	1.64	1.19	0.72	0.42
20°.........	2.06	1.60	1.77	1.54	1.06	0.83
40°.........	2.26	1.88	2.08	2.00	1.64	1.63
50°.........	2.44	2.12	2.32	2.32	2.07	2.28
60°.........	2.67	2.46	2.64	2.72	2.68	3.21
80°.........	3.41	3.66	3.60	3.88	4.77	6.41

In the neighborhood of phase-angle 50° the values resulting from all the six laws are nearly the same. If Euler's so-called "law," which has no sound physical basis and entirely fails to represent any observed facts, is ignored, the constant 2.20 represents the other five values of $\dfrac{q}{\phi(\alpha)}$ with an average deviation of only 4.6 per cent, and a maximum error of 6.8 per cent. If, therefore, M_{50} represents the ratio of the brightness of the planet at phase-angle 50° to that of the sun, the equation

$$A=2.20\ M_{50}\frac{R^2\Delta^2}{r^2} \tag{8}$$

will give the albedo, according to Bond's definition, within a few per cent, whatever may be the law of variation with phase, within the range of all those laws which have so far been derived from observation or seriously suggested theoretically.

Mars can be observed up to a phase-angle of 47°, and its albedo can therefore be determined with almost as much certainty as that of Venus or the moon. In the case of the brighter asteroids, which have been followed up to phase-angles between 20° and 30°, considerable extrapolation is necessary, and the results must be correspondingly uncertain. For Jupiter and the outer planets, the phase-angle can never be more than a few degrees, and attempts to determine their albedo must involve assumptions regarding the unknown laws of variation of their brightness in the unobservable phases. Since, however, the falling off in brightness with increasing phase-angle is certainly less for Jupiter and Saturn than for Venus, it is very probable that for these planets q is greater than 1.20. The assumption that q is 1.5, as would follow from Lambert's law, is probably within 15 per cent of the truth.

II. THE BRIGHTNESS OF THE EARTH-SHINE, AND THE STELLAR MAGNITUDE OF THE EARTH

The brightness of the earth, as seen from another planet, can be determined observationally only by measures of the illumination of the moon by the light reflected from the earth. The only recent series of observations of this, and doubtless the best, are these of Very,[1] who measured, with a photometer of his own design, the relative surface brightness of the sunlit portion of the moon, the portion lit by earth-shine, and the neighboring sky, the true brightness of the earth-shine being the difference of the last two quantities. His observations are summarized in Table II. The first four columns are taken from his paper, and give (1) the date of observation, (2) the elongation of the moon, which is the phase-angle of the earth at the moment, and is taken as negative for the waning moon, (3) the observed ratio of the surface brightness of the moon to that of the sky, and (4) that of the sunlit part of the moon to the earth-shine, after correction for the light of the sky. In the follow-

[1] *Astronomische Nachrichten*, **196**, 269–290, 1912.

ing columns, which are added by the writer, and contain a new reduction of Very's observations, appear (5) the ratio of the brightness of the sky to that of the earth-shine, (6) the difference in surface brightness between the sunlit and earth-lit portions of the moon, expressed in stellar magnitudes, (7) the ratio of the mean surface brightness of the moon at this phase to that of the full moon, expressed in magnitudes and taken from Table III of the preceding paper,[1] (8) the correction to the observed intensity of the earth-shine to reduce to mean distance, (9) the deduced ratio of the surface brightness of the earth-shine at mean distance and the observed phase to that of the full moon, which is obviously the difference of magnitude between the sun and the earth at mean distance and the given phase, as seen from the moon. The resulting

TABLE II

1	2	3	4	5	6	7	8	9
Sept. 28, 1911	70°	52	549	10.6	6.85	+2.01	−0.10	8.76
30	96	3095	4596	1.5	9.15	+1.48	− .11	10.52
Oct. 2	118	1149	3313	2.9	8.80	+1.10	− .10	9.80
26	54	3033	2889	0.9	8.65	+2.42	− .11	10.96
29	87	3626	14540	4.0	10.40	+1.67	− .10	11.97
Nov. 16	− 52	1871	2290	1.2	8.40	+2.39	+ .02	10.81
17	− 40	8579	2735	0.3	8.59	+2.65	.00	11.24
27	78	1358	1932	1.4	8.22	+1.86	− .08	10.02
Dec. 14	− 69	9380	2224	0.2	8.42	+2.08	.00	10.52
Feb. 14, 1912	− 42	10164	5769	0.6	9.40	+2.61	− .11	11.90
20	31.5	2476	698	0.4	7.11	+3.10	−0.02	10.19

values of the difference of magnitude between the earth and sun are too much influenced by the errors of observation, which are unavoidably large, to permit of an independent deduction of the law of variation of the earth's brightness with phase. To determine what influence various assumptions regarding this would have on the results, they have here been reduced on four different hypotheses, assuming for the law of the earth's variation in brightness with phase the Lommel-Seeliger law, Lambert's law, and the empirical laws found by Müller for Venus and in the preceding paper for the moon. The results are given in Table III, which gives, for the observations in order as above, and for each phase

[1] *Astrophysical Journal*, 43, 114, 1916.

law, (1) the correction to "full earth," (2) the deduced difference in magnitude between the mean full earth and the sun, and (3) the same quantity after correction for the systematic error described below. Upon grouping the results according to the relative brightness of the sky and earth-shine at the time of observation, as is done in the lower part of the table, it becomes evident that this greatly influences the measures, the computed brightness of the earth increasing with the brightness of the sky.

The observations of September 28, a hazy night, when the sky averaged ten times as bright as the earth-shine, and varied in brightness during the measures by four times the amount of the latter, should be entirely rejected. The other three groups can be brought into fair agreement on the assumption that the brightness of the sky-illumination at the point where the combined effect of it and the earth-shine was measured was, on the average, greater by 20 per cent than at the points outside the limb where it alone was measured. Although care was taken by Very to measure the light of the sky "at distances from the bright part of the moon comparable to those at which the earth-shine was measured," this assumption does not appear unreasonable when it is noticed that on October 26 ("sky clear") it was found that the sky immediately adjacent to the bright limb was ten times brighter than at the points where measures were made outside the dark limb. A very small difference in the distances from the bright part of the moon of the points at which the brightness of the sky and the earth-shine were measured would therefore explain the observed variations. On this hypothesis, the tabular results for the brightness of the earth-shine should be multiplied by the factor $1-0.20x$, where x is the measured ratio of the brightness of the sky to that of the earth-shine. The last column under each heading in Table III gives the difference of magnitude of the sun and full earth resulting from each night's measures, after application of this correction. The group-means given in the table show that the systematic difference depending on the brightness of the sky has disappeared, though the mutual accordance of the individual determinations is not improved. In forming final means, the observations made when the sky was more than twice as bright as the earth-shine have been

given half-weight. The average deviation of a single observation reduced to unit weight is given in the table. The magnitude of these quantities is due to the extreme difficulty of the observations. As a basis for further discussion, the mean of the values obtained with and without the systematic correction here suggested has been adopted. Taking the mean of the results of the first three assumptions concerning the variations with phase, which represent the observations equally well, and better than the fourth, it may be

TABLE III

Sky: Earth-Shine	Lommel-Seeliger			Lambert			Venus			Moon		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
10.6...	−0.8	(8.0)	.....	−0.9	(7.9)	.....	−1.2	(7.5)	.....	−1.9	(6.9)	.....
1.5...	−1.2	9.3	9.7	−1.4	9.1	9.5	−1.7	8.9	9.3	−2.7	7.9	8.3
2.9...	−1.8	8.0	8.9	−1.9	7.9	8.8	−2.3	7.5	8.6	−3.7	6.1	7.2
0.9...	−0.4	10.5	10.7	−0.4	10.5	10.7	−0.8	10.2	10.5	−1.2	9.8	10.0
4.0...	−1.0	11.0	12.7	−1.1	10.9	12.6	−1.4	10.6	12.3	−2.3	9.7	11.4
1.2...	−0.4	10.4	10.7	−0.4	10.4	10.7	−0.7	10.1	10.4	−1.2	9.6	9.9
0.3...	−0.2	11.0	11.1	−0.2	11.0	11.1	−0.5	10.7	10.8	−0.9	10.3	10.4
1.4...	−0.8	9.2	9.5	−0.9	9.1	9.4	−1.2	8.8	9.1	−2.0	8.0	8.3
0.2...	−0.6	9.9	9.9	−0.7	9.8	9.8	−1.0	9.5	9.5	−1.7	8.8	8.8
0.6...	−0.3	11.6	11.7	−0.3	11.6	11.7	−0.6	11.3	11.4	−0.9	11.0	11.1
0.4...	−0.2	10.0	10.1	−0.2	10.0	10.1	−0.4	9.8	9.9	−0.7	9.5	9.6
Means	Obs.											
3.4...	2	9.5	10.8	.....	9.4	10.7	.....	9.0	10.4	.....	7.9	9.3
1.2...	4	9.9	10.2	.....	9.8	10.1	.....	9.5	9.8	.....	8.8	9.1
0.4...	4	10.6	10.7	.....	10.6	10.7	.....	10.3	10.4	.....	9.9	10.0
General mean		10.15	10.47	.....	10.10	10.41	.....	9.81	10.13	.....	9.20	9.54
Average deviation..		± 0.72	0.75	.....	± 0.74	0.80	.....	± 0.75	0.79	.....	± 0.95	0.95
Adopted.....	10.31			10.25			9.97			9.37		

estimated that the mean full earth, as seen from the moon, is $10^{M}.2$ fainter than the sun, and of magnitude −16.5, or 40 times brighter than the full moon appears to us. The magnitude of the earth as seen from the sun would then be −3.5; and as seen from Venus when nearest, −6.3. The probable error of these values must be at least $\pm 0^{M}.25$, or over 20 per cent. and arises more from the errors of the observations than from any uncertainty in the assumptions made during the reductions.

Professor Very's own reduction of his observations leads to substantially the same value for the brightness of the full earth. From his data, combined with certain results of Arago, he concludes that, when the moon is at quadrature, the sunlit half of its disk is 5100 times brighter than that illuminated by the earth-shine. But at this time the mean surface brightness of the moon is 0.16 times that of the full moon, according to Very's reduction-curve, while, assuming the earth to behave like Venus, its light is 0.28 that of the full earth. Hence the ratio of sunlight to full earth-light on the moon is $5100 \times \frac{0.28}{0.16}$, or 9000, corresponding to a difference of magnitude of 9.89, which agrees within $0^{M}.1$ with the value here derived on the assumption of the same phase law.

It will, however, be shown later that the value of the earth's albedo which Professor Very deduces from his observations is about twice too great.

In a later paper[1] Very gives the results derived by him from measures of spectrograms of the earth-lit and sunlit portions of the moon, obtained by Slipher at the Lowell Observatory. His conclusions regarding the relative intensity of the light of these two sources depend on assumptions regarding the photographic action of exposures of very different durations to light of different brightness. For example, on January 3, 1911, one spectrogram was obtained of the earth-shine, with exposure 2520 seconds, and two of the sunlit moon, with exposures of 16 and 4 seconds. According to Very's measures, the mean ratio of the intensity of the lunar spectrogram of 16^{s} exposure to that of the earth-shine (corrected for diffuse light from the sky) is 7.109, while the corresponding ratio for the 4^{s} exposure is 3.913. On the assumption that the photographic intensity was proportional to the product of the intensity i of the light by the time t of exposure, the first of these comparisons would make the sunlit moon 1120 times as bright as the earth-shine, and the second 2460. Very concludes from these and similar figures that "the Cramer instantaneous isochromatic plates are more sensitive than the product $i \times t$, when the exposure is to a light as feeble as that of the earth-shine spectrum, enduring for upward

[1] *Astronomische Nachrichten*, **201**, 353–399, 1915.

of an hour, and less sensitive than $i \times t$, when the illumination approaches that of the lunar spectrum, with a duration of a few seconds only," and hence that, if the values of the relative brightness of the moon and earth-shine obtained as above are plotted against the exposures on the moon and a curve drawn to represent them, and extended to give the ratio which would have been obtained for an indefinitely short exposure on the moon, this will be the correct value.

On this hypothesis, the shortest exposures on the moon should give the best results, but even they will make the brightness of the earth-shine come out too great. Acting on this principle, Very's results from four nights' spectrograms have been reduced anew in the same manner as his visual observations. The details are given in Tables IV A and IV B. Table IV A contains data derived

TABLE IV A

Date	Exposures		Ratio of Sky to Earth-Shine	Ratio of Photographic Intensities	$M:E$
	Earth-Shine	Moon			
1911					
Jan. 3.......	2520^s	4^s	0.9	3.91	2460
4.......	4800	2	3.3	1.39	3340
	....	2	....	1.18	2830
1912					
Aug. 8.......	4800	1	0.2	1.36	3529
9.......	4260	1.5	0.1	1.06	2674

TABLE IV B

Elongation	Difference Magnitude	Reduction to			$S:E$
		Full Moon	Full Earth	Mean Distance	
+39°.3......	8.48	+2.83	−0.55	−0.01	10.75
+51.2......	8.81	+2.49	−0.74	+0.01	10.57
	8.63				10.39
−47.8......	8.87	+2.48	−0.69	+0.08	10.74
−34.0......	8.57	+2.83	−0.47	+0.10	11.03

from Very's paper, and gives, in successive columns, the date of observation; the exposure on the earth-shine; the shortest exposure on the moon; the ratio of the brightness of the sky to that of the earth-shine (taken from Very's tables [*op. cit.*] on pp. 374, 375, and

379); the ratio of the photographic intensity of the lunar spectrum to that of the earth-shine (after correction for the light of the sky); and finally the ratio $M:E$ of the surface brightness of the sunlit part of the moon to that of the earth-lit portion, computed as described above. Table IV B gives, for the observations in order, the elongation of the moon; the difference in brightness between the sunlit and earth-lit portions, reduced to stellar magnitudes; the correction for the difference in surface brightness between the full moon and the moon at the given phase, taken from Table III of the preceding paper; the reduction to full earth, on the assumption that its phase-variations follow the same law as in the case of Venus; the reduction to mean distance of the earth and moon; and finally the ratio $S:E$ of the light of the sun and mean full earth, expressed in stellar magnitudes.

The mean of these last quantities, giving each night equal weight, is 10.75, as against 9.97 derived from the visual observations on the assumption of the same phase law. The photographic observations therefore make the earth-shine only half as bright as do the visual observations. This is just what might be expected if the plates had followed the ordinary law for faint illumination and long exposure, and been "less sensitive than $i \times t$." This was certainly the case for the exposures to the lunar spectrum. As only one exposure on the earth-shine was made on each night, the measures afford no direct means of determining whether it was so in this case too. It seems reasonable, however, to assign very little if any weight to these observations, for the present purpose, in comparison with the visual measures.

Professor Very concludes that the photographic measures are in excellent agreement with the visual observations, because they give values of the ratio $M:E$ which agree well with a curve previously derived from his visual measures.[1] But this curve is not consistent with his assumptions with regard to the phase-variations of the earth and moon. For elongations of 30°, 50°, and 90° it gives the values of $M:E$ as, respectively, 2450, 3150, and 5100. According to Very's curve for the surface brightness M of the moon, its values at these elongations are 0.08, 0.092, and 0.16.

[1] *Astronomische Nachrichten*, **196**, 274, 1912 (Fig. 3).

This demands that the brightness E of the earth-shine at these phases should be in the ratios 1.04, 0.92, and 1.00; that is, that the earth's light should be as bright at quadrature as when it was nearly full. If the earth follows the same law as Venus, these ratios should be 2.74, 2.05, and 1.00. Hence Very's curve makes the earth-shine more than twice as faint, in proportion to the sunlit region, at the elongations at which observations were made, as his theoretical assumptions indicate.

Very's visual color-estimates, Slipher's spectrograms, and Tikhoff's photographs[1] agree in showing that the earth is bluer than ordinary moonlight. All three observers explain this on the assumption that a considerable part of the light reflected from the earth is scattered by its atmosphere, and is blue like the sky. Tikhoff has derived provisional numerical results from his photographs, and concludes that the ratio of the light scattered by the molecules of the air, which varies inversely as the fourth power of the wave-length, to that reflected by clouds, dust, and the earth's surface is 0.18 at λ 6400, 0.32 at λ 5650, and 1.28 at λ 3750. Taking the mean effective visual wave-length at 5200, for this very faint light, and the photographic wave-length as 4200, it would follow that the earth's photographic albedo should be about 40 per cent greater than its visual albedo. The reflecting power of the earth for the sun's radiation as a whole, according to a summary calculation from Abbot's energy-curve, should be the same as for λ 5400, and therefore practically equal to the visual albedo.

III. CONCLUSIONS AND SUMMARY

It may be well at this point to summarize the results of the discussion, including both the present paper and its predecessor.

1. Bond's definition of albedo, as the ratio of the whole amount of light reflected in all directions from a sphere illuminated by parallel rays to the amount of light incident on the sphere, appears to be the most suitable for astronomical purposes.

2. The albedo A of any planet, according to this definition, may be expressed as the product of two factors, of which one, q, depends only on the law of variation of its light at different phases, while

[1] Pulkowa, *Mittheilungen*, **6**, 22, 1914.

the other, p, depends on the brightness at full phase and the geometrical relations. The factor p may be defined verbally as the ratio of the observed brightness of the planet at full phase to that of a flat disk of the same size and in the same position, illuminated and viewed normally, and reflecting all the incident light in accordance with Lambert's law, or algebraically by equations (5) or (6). The factor q is defined by equation (7) and may be found by mechanical quadrature when the law of variation of brightness with phase is known. Its value varies from 1.635 for the theoretical law of Lommel and Seeliger to 0.420 in the case of an empirical law found from the observations of Mercury. In all the cases, theoretical or observed, which demand practical consideration, it may be found within a few per cent by multiplying the ratio of the brightness at phase-angle 50° to that at full phase by 2.20.

3. The visual stellar magnitude of the sun, on the Harvard scale, is -26.72 ± 0.04, according to the mean of the observations of Zöllner, Fabry, Ceraski, and W. H. Pickering, which were made by different methods and are in excellent agreement. The photographic magnitude, according to the observations of King and Birck, is -25.93, and its color-index $+0^{M}.79$, agreeing within the errors of observation with the average for stars of Class G.

4. Müller's observations of the brighter planets require a correction of approximately $-0^{M}.06$ to reduce them to the Harvard scale, while that for Uranus and Neptune is insensible. King's photographic observations, combined with Müller's visual observations at the same phases, give for the color-indices on the scale of King's system: Venus, $+0^{M}.78$; Mars, $+1^{M}.38$; Jupiter, $+0^{M}.50$; Saturn, $+1^{M}.12$.

5. The results of the seven observers who have determined the variations of the moon's brightness with phase are all in satisfactory agreement with the mean light-curve, which is given in Table III of the preceding paper,[1] and may be regarded as very well determined. The full moon is 8.7 times brighter than the first quarter, and 10.0 times brighter than the last quarter.

6. The results of different observers for the stellar magnitude of the mean full moon are discordant. Herschel's observations

[1] *Astrophysical Journal*, 43, 114, 1916.

have been reduced with modern photometric magnitudes of his comparison stars, and an error which had crept into the earlier reduction corrected. The visual magnitude adopted as the weighted mean of those observations not obviously affected by systematic error is -12.55, with a probable error of $\pm 0^{M}.07$, as derived from the observations, but very likely greater in reality. The photographic magnitude, -11.37, has been well determined by King. The resulting color-index, $+1^{M}.18$, is in accordance with the spectro-photometric work of Wilsing and Scheiner, which shows that moonlight is decidedly redder than sunlight.

7. The intensity of sunlight from the zenith is 103,000 meter-candles, and that of mean full moonlight 0.24 meter-candle. A standard candle, if of approximately the same color as the stars, would appear of magnitude -14.18 at a distance of 1 m, and $+0.82$ at 1 km.

8. Very's observations of the intensity of the earth-shine indicate that the mean full earth, as seen from the moon, is forty times brighter than the full moon as seen from the earth, and that the stellar magnitude of the earth as seen from the sun is -3.5, with an uncertainty of at least 25 per cent, or $0^{M}.20$. Owing to the blue light reflected by the sky, the photographic albedo of the earth is probably about 40 per cent greater.

Collecting now the numerical data, the results are exhibited in Table V (for the objects named in the first column) as follows: (1) the stellar magnitude at mean opposition, m_0 (for Mercury and Venus, the magnitude at full phase, mean distance from the sun, and unit distance from the earth); (2) the magnitude g which it would have at full phase and unit distance from the earth and sun; (3) the assumed mean semi-diameter σ_1 at unit distance; (4) the quantity p, computed by the equation (6) p. 177; (5) the factor q defined above; (6) the albedo A, according to Bond's definition; (7) the color-index (when known); and (8) the photographic albedo, according to Bond's definition.

Two sets of values are given for Mercury, the first corresponding to Müller's linear empirical formula for the phase-variations and the second to his quadratic formula.[1] The data for Saturn refer

[1] *Astrophysical Journal*, **43**, 108, 1916.

to the ball of the planet, with ring invisible. The results obtained for the earth from Very's visual observations are reduced on all four assumptions regarding the law of variation with phase. The values of q for Mercury, Venus, and the moon have been derived by mechanical integration from the observed laws of variation with phase, and are accurate. Those for Mars and the asteroids have

TABLE V

Object	m	g	σ_1	p	q	Visual Albedo A	Color-Index	Photographic Albedo
Moon	−12.55	+0.40	2″.40	0.105	0.694	0.073	+1.18	0.051
Mercury	− 2.94	−0.88	3.45	.164	0.42	.069	......	......
	− 2.12	−0.06	3.45	.077	0.72	.055	......	......
Venus	− 4.77	−4.06	8.55	.492	1.20	.59	+0.78	.60
Mars	− 1.85	−1.36	4.67	.139	1.11	.154	+1.38	.090
Jupiter	− 2.29	−8.99	95.23	.375	1.5:	.56:	+0.50	.73:
Saturn	+ 0.89	−8.67	77.95	.420	1.5:	.63:	+1.12	0.47:
Uranus	+ 5.74	−6.98	36.0	.42	1.5:	.63:	......	......
Neptune	+ 7.65	−7.06	34.5	.49	1.5:	.73:	......	......
Ceres	+ 7.15	+3.70	0″.53	.10	0.55:	.06:	......	......
Pallas	+ 7.84	+4.38	0.34	.13	0.55:	.07:	......	......
Juno	+ 8.95	+5.74	0.14	.22	0.55:	.12:	......	......
Vesta	+ 6.04	+3.50	0.27	.48	0.55:	.26:	......	......
Jupiter I	+ 5.54	−1.16	2.38	.46	1.5:	.69:	......	......
II	+ 5.69	−1.01	2.08	.51	1.5:	.76:	......	......
III	+ 5.08	−1.62	3.62	.30	1.5:	.45:	......	......
IV	+ 6.26	−0.44	3.49	.11	1.5:	.16:	......	......
Titan	+ 8.30	−1.26	2.9	0.33	1.5:	0.50:	......	......
THE EARTH (FROM VERY'S VISUAL OBSERVATIONS)								
Lommel-Seeliger	......	−3.46	8.79	0.27	1.64	0.45	+0.45:	0.6:
Lambert	......	−3.52	8.79	.29	1.50	.43	......	......
Like Venus	......	−3.80	8.79	.37	1.20	.45	......	......
Like Moon	......	−4.40	8.79	0.65	0.70	0.45	......	......
THE EARTH (FROM VERY'S REDUCTION OF SLIPHER'S SPECTROGRAMS)								
Like Venus	......	−3.02	8.79	0.18	1.20	0.22	......	......

been determined from the loss of brightness at phase-angle 50°. In the former case this should give a nearly correct value. For the asteroids considerable extrapolation is necessary, and the tabular values, which have been computed with the mean law of variation for all the asteroids which have been observed, are considerably uncertain. The value $q=1.5$ which has been assumed for the outer planets, in the impossibility of observing them except very near the full phase, may be in error by 15 per cent. The same

value has been adopted for the satellites of Jupiter and Saturn, in the total absence of observational information. If these small and presumably atmosphereless bodies behave like Mercury, Mars, or the moon, the actual values of q for them would be much smaller, and the tabular values of A may in these cases be regarded as upper limits.

Concerning the diameters of the various bodies which are employed in the reductions, it may be briefly stated that they have been taken from the sources which appeared most likely to be free from systematic error, especially that arising from irradiation. Thus Sampson's values[1] for Jupiter and its satellites, derived from the Harvard eclipse observations, and Abetti's small value for Neptune,[2] corrected for irradiation have been adopted, while the adopted diameters of Venus and Mercury are greater than those given by the measures during transits. Barnard's diameters[3] for the four asteroids are used. The most remarkable value, $0''.54$ for Vesta at unit distance, is confirmed by the interferential measures of Hamy,[4] who finds $0''.53$.

In conclusion, a few remarks may be made upon the physical interpretation of these results.

For such purposes as the computation of the probable temperature of a planet, the value of the albedo given in the column headed A should unquestionably be used, as it represents the amount of solar light which is actually reflected from the planet. When attempting to estimate the diameters of the fainter asteroids and satellites, on the basis of their observed magnitudes at opposition, the quantities p, which are independent of the peculiarities of the phase-variation of the planets, should obviously be employed. It is easy to see that the values p should also be used for comparison with the observed reflecting powers of terrestrial substances. It has been recognized since Zöllner's time that the rapid decrease in brightness of the moon with phase arises mainly from the rough character of its surface, the shadows of the irregularities being

[1] *Harvard Annals*, **52**, 333.

[2] *Memorie della Società degli Spettroscopisti Italiani* (2), **1**, 113, 1912.

[3] *Astronomische Nachrichten*, **157**, 262, 1902.

[4] *Comptes Rendus*, **128**, 851, 1899.

invisible to us at the full, but covering more and more of the surface with advancing phase. Zöllner concluded that the observed variation could be accounted for by supposing that the whole surface of the moon was covered with mountains having slopes that were inclined at an angle of 52° to the horizontal, but his analysis suffers from an error pointed out by Searle.[1] The effect is too great to be explained by the shadows of the visible lunar mountains, but the very plausible assumption that a great part of the moon's surface is covered by broken fragments of rock, in whose interstices innumerable shadows are formed, appears sufficient, recalling Seeliger's theory of Saturn's rings, in which the variation at small phase-angles is still more pronounced. Very little of the solar radiation which penetrated into a crevice between two such rocks would get back again into space, for most of that which was diffusely reflected from the region where the sunbeam struck would be caught on the walls of the crevice. Hence a rough surface of this sort would be a much better absorber of incident radiation than a smooth, diffusely reflecting surface of the same material. At the full phase, however, when our line of sight reaches right down the path of the sun's rays, we shall see the sunlit patches at the bottom of the crevices, and the surface will appear just about as bright as a smooth surface of the same material. Hence there is good reason to believe that if the moon's surface could be smoothed out, without otherwise altering its properties, the brightness of the full moon, and the value of p deduced therefrom, would be but little changed, while the brightness at large phase-angles, and hence the values of q and A, would be considerably increased. For the outer planets, however, which appear considerably brighter at the center than at the limb, the values of A may be nearer the true reflecting power of the surface than those of p.

Wilsing and Scheiner[2] have determined the reflecting power of many ordinary rocks, using an approximately flat, rough, natural surface normal to the incident and reflected rays. Their formula of reduction gives exactly the quantity which has here been designated by p, and the conditions of observation appear very

[1] See Müller, *Photometrie der Gestirne*, p. 77.

[2] *Potsdam Publications*, **20**, Part IV, 1909.

similar to those which obtain in the case of the full moon. Their results for the albedo of a number of typical rocks are: "Liparit-bimsstein," 0.56; yellow sandstone, 0.38; red granite, 0.36; volcanic ash, 0.18; syenite, 0.13; trachyte-lava, 0.10; clay-slate, 0.07; basalt, 0.06. In almost all cases the reflecting power was greater for the red end of the spectrum than for the blue, the average difference being about 20 per cent.

Abbot,[1] from energy measurements, concludes that the albedo of a cloud surface is about 0.65, and, after allowance for atmospheric absorption, computes that the albedo of the earth, in substantially the sense of Bond's definition, would be 0.60 if it were completely covered with high clouds, 0.14 if it were cloudless, and 0.37 as matters actually are. These values are provisional, but no later ones have been published.

The results of the present discussion therefore entirely confirm the familiar views that the reflecting power of Venus and the outer planets is comparable with that which might be expected from cloud surfaces, while that of Mercury, Mars, and the moon, which have little or no atmospheres, is similar to that of ordinary rocks. The high albedo found for Neptune is somewhat uncertain, on account of the great difficulty of getting precise measures of the planet's diameter. The most remarkable features are the high values for Vesta and the first and second satellites of Jupiter—none of which can have much if any atmosphere—but even these can be matched by some terrestrial rocks.

It is also worthy of remark that the values of p for Venus, the earth, Vesta, Jupiter, Saturn, Uranus, and Neptune are higher than the theoretical limit, 0.25, for a body reflecting according to the Lommel-Seeliger law, and that, for all the bodies which have values of p below the limit, the known variations with phase are inconsistent with this law, except in the sole case of the fourth satellite of Jupiter, where there are no data.

The value of the earth's albedo found from Very's visual observations comes out almost the same on all the assumptions regarding the variations with phase, as it ought to do for observations made at phase-angles averaging not far from 50°. It is intermediate

[1] *Annals of the Smithsonian Astrophysical Observatory*, 2, 161–163.

between the values for the cloudless and cloudy planets, and agrees with Abbot's estimate within the error of the observations.

Professor Very, from the same observations, has derived the value 0.89 for the albedo of the earth. His reasoning is substantially as follows: The light of the full earth, as seen from the moon, is 1/9000 that of the sun (according to his own reduction, a result which differs little from that of the present paper). But, according to Zöllner, the light of the full moon is 1/618,000 that of the sun; and the angular area of the earth as seen from the moon is 13.4 times that of the moon as seen from the earth. Hence, if the earth were of the same albedo as the moon, its light at the full would be 1/46,120 of sunlight, and its actual albedo must be 5.12 times that of the moon. Zöllner's value for the albedo of the moon is 0.174; therefore that of the earth is 0.89.

As Guthnick has pointed out,[1] the trouble here lies with the value adopted for the albedo of the moon. Zöllner, in his original discussion,[2] distinguishes between the "apparent albedo" of the moon, which is that derived from the observations in the ordinary way by application of Lambert's law for a smooth, diffusely reflecting sphere, and the "true albedo," which depends on his assumption that the whole surface of the moon is covered with declivities of an average slope of 52°, the sides of which reflect diffusely according to Lambert's law. Under these highly artificial conditions, the slopes, receiving sunlight more obliquely than would a level surface, would appear less luminous at full moon, and a corresponding correction is necessary to find the "true albedo" which the moon would exhibit if these hypothetical irregularities were smoothed flat. Zöllner's "true albedo," 0.174, does not therefore represent the actual intensity of the light of the full moon. His "apparent albedo," 0.120, as can easily be verified, corresponds to his observed ratio of moonlight to sunlight, and to Lambert's law. The corresponding value of p, in the notation of the present paper, is 0.080. Hence, what Very's argument really shows is that the value of p for the earth is 0.080×5.12, or 0.41. On his assumption that the law of variation with phase is the same

[1] *Astronomische Nachrichten*, **198**, 253, 1914.

[2] *Photometrische Untersuchungen*, p. 162.

as for Venus, this must be multiplied by $q=1.20$ to get the actual albedo, which thus comes out 0.49. This is about 10 per cent higher than the value found above, the difference arising from the difference between the values deduced by Very and by the writer for the brightness of the full earth, but being less than the probable error of either.

It appears, therefore, that the value of the earth's albedo resulting from Professor Very's measures of the earth-shine, far from being inconsistent with Abbot's value of the solar constant (1.93 calories), is actually in agreement with it, within the unavoidable uncertainty of the photometric measurements.

In conclusion, the attention of photometric observers may be called to the uncertainty that still exists concerning some of the most fundamental data. Further and more accurate observations, by visual and photo-visual methods, of the relative brightness of the moon and stars, and of the standard candle and the stars, and observations of the earth-shine, both visual and photographic, are especially desirable.

ADDENDUM

Mr. Evershed, in a recent letter to *Nature*,[1] calls attention to the fact that the disk of the moon, when rising or setting behind distant snow-clad mountains, appears very similar in color and brightness to the snow, and queries whether this observation is consistent with the low albedo ordinarily attributed to the moon, especially when it is considered that the rays reaching the observer from the snow and the moon have traversed approximately equal thicknesses of the atmosphere. His observations regarding the moon's appearance under these circumstances are confirmed by the writer's very vivid recollections of sunrise at Taormina, with the moon setting behind the snowy slope of Etna. On that occasion, however, the moon, though as white as the snow, was hardly as bright, seeming in comparison almost unsubstantial, like a bubble.

Yet the photometric observations summarized above show beyond a doubt that the albedo of the moon is actually very low—probably less than one-fifth of the value for a snow-clad satellite—and add to the difficulty by indicating that the moon's light is decidedly yellowish.

The solution of the problem is probably to be found in two facts: (1) In comparing the moon, seen in daylight, with distant snows, the comparison is instinctively made between the high lights in both areas, that is, between the brightest parts of the disk and the fully illuminated snow, while the dark

[1] *Nature*, **96**, 369, 1915.

maria are compared with the shadows on the snow. As the albedo of these bright areas is much greater than the average for the moon's disk as a whole, this diminishes the difficulty, though it does not by itself remove it. (2) When the moon is seen by daylight, most of the light which appears to come from it comes in reality from the illuminated air between the observer and the moon, and relatively little from the moon itself. A glance at the half-moon by daylight shows that the *maria* are conspicuously blue, while the phase limb is hardly visible, the light of the sky overpowering the faint illumination of this part of the lunar surface. With even a low telescopic power these effects are greatly exaggerated. According to Kimball's measurements, the diffused light of the sky at noon, even on a clear day, illuminates a horizontal surface about one-tenth as strongly as the direct rays of the sun. It follows that the mean surface brightness of the noonday sky is nearly equal to that of the full moon, and therefore several times greater than that of the half-moon. When the moon and sun are both low in the sky, their light is much diminished by atmospheric absorption. That of the sky is also less than with a high sun, but not so greatly weakened, as is obvious without instrumental aid to anyone who compares the illumination of a vertical surface by the rays of the setting sun with that of a neighboring horizontal surface by the sky.

When the moon is rising or setting, with the sun above the horizon, the light which comes from the sky in front of it must therefore be considerably greater than that which really comes from the moon, and it must appear at least two or three times brighter, and also much whiter, than it would if rising, under otherwise identical circumstances, during the night.

The combination of these two factors appears to be sufficient to explain Mr. Evershed's observations.

PRINCETON UNIVERSITY OBSERVATORY
January 20, 1916

ON THE TEMPERATURE AND RADIATION OF THE SUN[1]

By FELIX BISCOE

I. OBSERVATIONS WITH THE SPECTRO-BOLOMETER

The purpose of this part of the paper is the determination of the temperature of the sun from the intensity of radiation for individual wave-lengths in its spectrum. The observations at the Smithsonian Astrophysical Observatory[2] at Washington will be employed here.

1. *The intensities of radiation of the sun outside the earth's atmosphere.*—The observations mentioned above were principally made at Mount Wilson and Mount Whitney in 1909–10, the rays of the whole visible solar surface being employed. The coefficients of transmission of the earth's atmosphere were determined by the secant law. In the mean from the whole observational material the intensities I of the solar radiation shown in Table I were obtained for the different wave-lengths λ in μ in the normal spectrum, outside the earth's atmosphere, expressed in arbitrary units.

TABLE I

λ	I	λ	I	λ	I
0.300	539	0.430	5321	0.800	2665
0.325	1271	0.450	6027	1.000	1657
0.350	2684	0.470	6240	1.300	898
0.375	3459	0.500	6062	1.600	532
0.390	3614	0.550	5623	2.000	247
0.400	4338	0.600	5042	2.500	43
0.420	5251	0.700	3644	3.000	14

For the extreme values of the wave-lengths in this table the intensities I are affected with some uncertainty. The value of the solar constant, that is, the quantity of heat which outside the

[1] Extract from the author's paper in the *Warsaw University News*, 1915 (in Russian). See also the author's paper in *Astronomische Nachrichten*, **183**, 241, 1910, entitled "Die Temperatur der Sonne."

[2] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **3**, 196–201, 134, and 157, 1913.

earth's atmosphere the sun radiates upon the square centimeter of the earth with vertical incidence per minute, reduced to the mean distance of the earth, is determined to be

$$S = 1.932 \text{ gm cal.}$$

2. *Determination of the coefficients of transmission of the solar atmosphere as well as the intensities of radiation of the solar photosphere.*—That part of the observations mentioned which concerns itself particularly with the decrease in intensity on the sun's disk toward the edge was chiefly made in 1907 at Washington. From the mean of many observations which were made at different degrees of transparency of the earth's atmosphere, as well as at different zenith distances of the sun, there were derived for the selected wave-lengths λ at definite fractions ρ of the solar radius from the center, the accompanying ratios of intensity (Table II) expressed in units of the intensities at the center of the sun's disk.

TABLE II

λ \ ρ	0.00	0.20	0.40	0.55	0.65	0.75	0.825	0.875	0.92	0.95
0.323	1.0000	0.960	0.897	0.835	0.775	0.690	0.660	0.530	0.452	0.382
0.386	1.0000	.9801	.9258	.8556	.7920	.7097	.6326	.5543	.4826	.4177
0.433	1.0000	.9780	.9271	.8663	.8060	.7290	.6471	.5827	.5098	.4499
0.456	1.0000	.9857	.9416	.8848	.8314	.7564	.6810	.6160	.5377	.4706
0.481	1.0000	.9871	.9438	.8914	.8401	.7706	.7007	.6378	.5658	.4988
0.501	1.0000	.9850	.9454	.8945	.8449	.7773	.7106	.6501	.5826	.5171
0.534	1.0000	.9870	.9499	.9018	.8561	.7919	.7282	.6720	.6053	.5478
0.604	1.0000	.9894	.9568	.9129	.8722	.8160	.7606	.7102	.6485	.5935
0.670	1.0000	.9906	.9612	.9241	.8870	.8376	.7860	.7404	.6803	.6292
0.699	1.0000	.9902	.9629	.9261	.8903	.8412	.7925	.7476	.6909	.6371
0.866	1.0000	.9922	.9690	.9388	.9108	.8711	.8302	.7918	.7440	.6987
1.031	1.0000	.9978	.9774	.9508	.9251	.8886	.8507	.8159	.7724	.7298
1.225	1.0000	.9948	.9756	.9530	.9316	.9007	.8654	.8336	.7944	.7565
1.655	1.0000	.9965	.9820	.9657	.9504	.9275	.9012	.8770	.8471	.8155
2.097	1.0000	0.9965	0.9858	0.9709	0.9563	0.9361	0.9149	0.8924	0.8664	0.8377

The values given in Table I furnish us with the mean intensities of the radiation of the whole visible solar surface. For our further investigations we need, however, the values for radiation of the whole visible solar surface, but of such intensity as is found at the center of the sun's disk, the same to be expressed for each wavelength in the same units as in Table I. In order to accomplish this,

we draw on the axis of abscissas, at the fraction ρ of the solar radius R, the ordinates y of the magnitude of the relative intensities of Table II, which we assume to be valid up to the middle points of the adjacent intervals of the abscissas. Then the entire intensity of radition of the visible solar surface in units of the intensity at the center will be expressed for each wave-length by the sum of the volumes of cylinders which result from rotating about the axis of ordinates of the separate rectangles. The formula for the volumes is

$$V_0=2\pi R\frac{0.0+0.1}{2}R(0.1-0.0)y_0,\ V_1=2\pi R\frac{0.1+0.3}{2}R(0.3-0.1)y_1,\ldots.$$

$$V_{10}=2\pi R\left(0.975+\frac{1-0.975}{3}\right)R\frac{1-0.975}{2}\frac{y_0}{2},$$

where in the last volumes the rotation of a triangle with the height equal to the half of the last ordinate is assumed. If we now divide the sum of these volumes by the base πR^2, we shall obtain for each wave-length, with the exception of the limiting values of Table II, the mean intensities m of the radiation on the visible solar surface in units of the intensity at the center of its disk given in Table III.

If we now divide the intensities I of Table I by the corresponding fraction m, we obtain for each wave-length the value of the radiation for the whole visible solar surface reduced to the intensity at the center of the disk.

On the assumption that the decrease in brightness on the solar surface is produced in the inappreciably radiating solar atmosphere itself, the coefficients of transmission of the solar atmosphere can be derived as follows from Table II:

If I is the intensity of radiation of the solar photosphere; I_0, the intensity outside the earth's atmosphere at the center of the visible solar disk; I_z, that at any point whatever in the apparent distance ρ from the center expressed in units of the solar radius; further, if z be the angle which the ray of light proceeding from this point and reaching the observer forms with the solar radius prolonged through this point; and ξ is the refraction of this ray in the solar atmosphere; then Laplace's theory of extinction yields the following equation:

$$\log\frac{I_z}{I}=-K\frac{\xi}{\sin z}=-K\alpha\sec z\,,$$

where K denotes a constant, and the refraction on the sun is expressed by $\xi = a \tan z$. From this expression it follows that

$$\frac{I_z}{I_0} = p^{\sec z - 1} \tag{1}$$

if the transmission coefficient $\frac{I_0}{I}$ is set equal to p.

From the fundamental equation of the theory of refraction applied to the points of a light-ray where it leaves the solar surface or the point of observation, there follows the relation:

$$\sin z = \frac{D \cdot \sin \sigma}{R \cdot \mu} = \frac{\sin \sigma}{\sin \sigma_0 \cdot \mu} = \frac{\rho}{\mu}, \tag{2}$$

if R is the radius of the sun, μ the coefficient of refraction at its surface, D the distance of the place of observation from the center of the sun, σ the apparent distance of the point of the solar surface

TABLE III

λ	m	I	f	T	λ	m	I	f	T
0.386	0.705235	3573	12437	7200	0.670	0.812568	4040	7420	7200
0.433	0.719581	5450	17077	7700	0.699	0.816577	3657	6560	7100
0.456	0.741503	6120	16538	7700	0.866	0.842530	2300	3702	6800
0.481	0.754034	6210	15605	7700	1.031	0.858651	1580	2370	6700
0.501	0.760813	6060	14573	7700	1.225	0.867743	1060	1538	6800
0.534	0.773816	5790	13058	7600	1.655	0.892505	480	632	7000
0.604	0.795179	5000	9820	7400	......	....	......	......	......

under consideration from its center, while σ_0 indicates the apparent radius of the sun, as it must appear without its atmosphere. For the quotient $\frac{\sin \sigma}{\sin \sigma_0}$ can therefore be substituted the apparent distance ρ of the point from the sun's center, expressed in units of the sun's radius. Thus (1) contains the two unknowns p and μ when we consider (2). If we denote by p_0 and μ_0 their approximate values, and if we let the true values be $p = p_0 + \delta p$ and $\mu = \mu_0 + \delta\mu$, then we obtain for the solution, after differentiating (1),

$$\sigma = -\frac{I_z}{I_0} + p_0^{\sec z_0 - 1} + (\sec z_0 - 1) p_0^{\sec z_0 - 2} \cdot \delta p - p_0^{\sec z_0 - 1} \frac{\log p_0}{0.4343} \cdot \frac{\sec^3 z_0 \cdot \sin^2 z_0}{\mu_0} \cdot \delta\mu .$$

Nine such equations were then formed according to Table II for each of the wave-lengths of Table III. The solution of these equations gave the most probable values for p and μ as they are represented in Table IV. After introducing these values in (2) and (1), a certain theoretical value $\frac{I_z}{I_0}$ is obtained, which differs from the empirical values of Table II in the sense O−C by small amounts which are further given in Table IV.

If we finally divide the intensities I of Table I, after they have been divided by the appropriate fractions from Table III, still further by the corresponding fractions p of Table IV, we obtain for each wave-length the intensity of radiation J of the whole solar photosphere directed toward us, which would appear equally bright all over except for the absorption in the solar atmosphere. The two operations mentioned were performed at once because at first the intensities I were interpolated from Table I, for which the values m and p are given in Tables III and IV, respectively. The results are contained in the fourth and ninth columns of Table III.

3. *A determination of the temperature of the solar photosphere from an application of the radiation formula for a perfect radiator.*—If S' is the solar constant corrected for absorption in the solar atmosphere; if $u=15'59''.6$, being the mean apparent radius of the sun, and if i is the specific intensity of radiation of an element of surface, then we obtain from the fundamental photometric laws for self-luminous bodies, as well as from the definition of the solar constant,

$$S'=60\, i\, \pi \sin^2 u\,.$$

From analogous values for the monochromatic solar constant S'_λ we obtain, for the intensity of radiation i_λ at the wave-length λ, the formula

$$i_\lambda=\frac{S'_\lambda}{60\,\pi\,\sin^2 u}\,.$$

The quantity S'_λ is here determined as follows. In Fig. 1 the energy-curves A and B are so constructed that to 1 mm along the axis of abscissas corresponds a range of wave-lengths

$$\delta\lambda=0.000001 \text{ cm}$$

TABLE IV

λ	p	μ	0.20	0.40	0.5	0.65	0.75	0.825	0.875	0.92	0.95
0.386	0.40737	1.07292	−0.0040	−0.0068	−0.0069	−0.0020	+0.0105	+0.0300	+0.0342	+0.0540	+0.0621
0.433	0.44353	1.06814	−0.0075	−0.0111	−0.0071	−0.0033	+0.0092	+0.0202	+0.0364	+0.0547	+0.0690
0.456	0.49906	1.05919	−0.0017	−0.0044	−0.0037	+0.0005	+0.0076	+0.0194	+0.0323	+0.0448	+0.0544
0.481	0.52776	1.05698	−0.0012	−0.0061	−0.0051	−0.0024	+0.0056	+0.0191	+0.0314	+0.0486	+0.0578
0.501	0.54657	1.05502	−0.0039	−0.0070	−0.0069	−0.0048	+0.0023	+0.0164	+0.0296	+0.0501	+0.0608
0.534	0.57299	1.05388	−0.0027	−0.0061	−0.0067	−0.0041	+0.0019	+0.0151	+0.0294	+0.0480	+0.0656
0.604	0.64035	1.04796	−0.0023	−0.0073	−0.0121	−0.0125	−0.0089	+0.0026	+0.0155	+0.0329	+0.0502
0.670	0.67003	1.04497	−0.0019	−0.0063	−0.0078	−0.0080	−0.0022	+0.0087	+0.0230	+0.0415	+0.0584
0.699	0.68265	1.04374	−0.0026	−0.0060	−0.0087	−0.0090	−0.0049	+0.0070	+0.0204	+0.0382	+0.0543
0.866	0.73741	1.04207	−0.0021	−0.0061	−0.0086	−0.0076	−0.0034	+0.0065	+0.0179	+0.0351	+0.0524
1.031	0.77639	1.03890	+0.0026	−0.0017	−0.0050	−0.0060	−0.0048	+0.0014	+0.0105	+0.0254	+0.0406
1.225	0.79488	1.03900	−0.0008	−0.0054	−0.0068	−0.0057	−0.0022	+0.0030	+0.0117	+0.0268	+0.0428
1.655	0.85077	1.03515	−0.0004	−0.0045	−0.0056	−0.0046	−0.0022	+0.0017	+0.0084	+0.0210	+0.0330

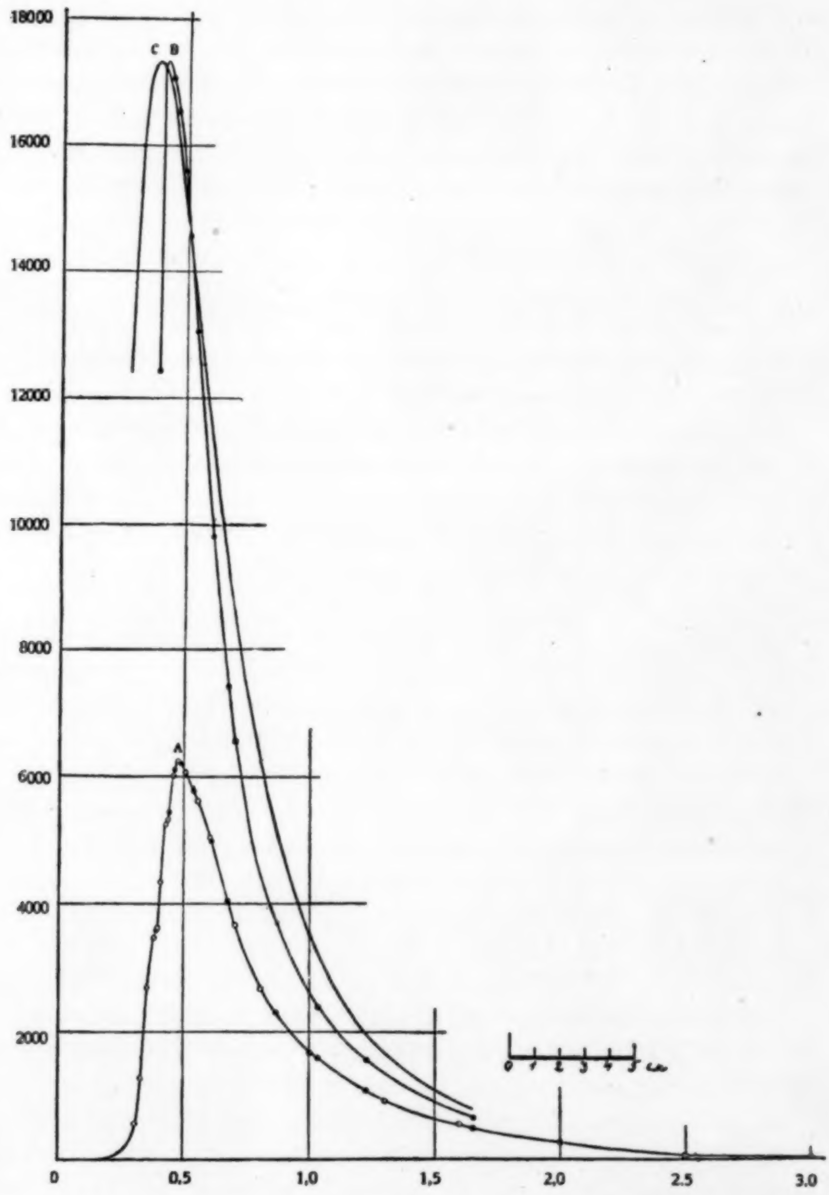
18000
16000
14000
12000
10000
8000
6000
4000
2000
C
B
A
0
0,5
1,0
1,5
2,0
2,5
3,0

FIG. 1

and in the axis of ordinates the arbitrarily assigned unit of the quantity I is that in Table I and of f in Table III. Then to the correct solar constant S' corresponds the total surface F' sq. mm, between the energy-curve B and the axis of abscissas, while to the quantity S for the interval of wave-length $d\lambda$ for a definite wave-length λ there corresponds an element of surface f sq. mm. Thence follows

$$S'_\lambda = \frac{S'}{F'} \cdot f \, ,$$

where the ratio $\frac{S'}{F'}$ is also equal to that for the original solar constant $S = 1.932$ gm cal., and the corresponding surface $F = 36{,}000$ sq. mm of the corresponding energy-curve A.

If we apply the formula for i_λ to an interval of 1 cm instead of the interval of wave-length $\delta\lambda = 0.000001$, we obtain

$$i_\lambda = \frac{f \cdot S}{d\lambda \cdot F \cdot 60\pi \sin^2 u} \, . \tag{1}$$

For the determination of the absolute temperature T for a perfect radiator, from its intensities of radiation between wave-lengths λ and $\lambda + d\lambda$, Planck has given the following formula

$$i_\lambda = \frac{c_1 \lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1} \, , \tag{2}$$

where the constants are

$$c_1 = 0.282 \cdot 10^{-12} \quad \text{gm cal. per sec per sq. cm}$$

and

$$c_2 = 1.4597 \text{ cm degrees.}$$

Thus (2) can be employed by comparison with (1) for the determination of the temperature of the solar photosphere. After substitution of the appropriate values there results the following numerical equation:

$$\frac{1.4597}{e^{\frac{c_2}{\lambda T}} - 1} = \frac{[0.699150 - 18]}{\lambda^5 \cdot f} \, ,$$

where for λ we substitute the wave-lengths in centimeters and for f the values from Table III. These absolute temperatures T of the

solar photosphere, determined from the separate wave-lengths λ, are collected in the fifth and tenth columns of Table III. The energy-curve of a perfect radiator of the absolute temperature 7700° C is represented in Fig. 1 by *C*.

4. *Discussion.*—It was assumed in this paper as a fact of observation that the sun is surrounded by a limiting stratum, the photosphere, which possesses not only an apparent but a material reality; also that the density of the photosphere from the limb to the deeper strata increases so rapidly that a comparatively small thickness of it is quite opaque; and that the surface of the photosphere radiates like a perfect black body. The gaseous strata lying above the photosphere, on account of their relatively small density and hence also on account of small temperature, can yield only a very nappreciable radiation in comparison with the photosphere; they reveal themselves, by a selective and general absorption of the photospheric radiation, as the solar atmosphere. On this assumption the coefficients of transmission of the solar atmosphere were derived, which appear entirely probable values, in respect to their absolute magnitude and their increasing progress with increasing wave-lengths. The indices of refraction derived therefrom have also a natural progression, decreasing with increasing wave-length; as to their absolute magnitudes, it may be remarked that in these is contained, not only the effect of the density on the absorption, which is taken into account, but also the effect of the height of the solar atmosphere, which operates similarly but which has not been taken into account; hence, the indices of refraction could only be smaller. The values of the unknown distribution of intensity on the solar surface derived from these values so far differ very little from that actually observed.

From the final intensity of radiation in the solar photosphere we obtain by the formulae for a perfect radiator the absolute temperature of the solar surface to be, on the average, $T_m = 7300° \pm 100°$ C. If we consider the great effect of the numerous corrections on the final intensities of radiation, as well as the uncertainties in the latter, particularly for longer wave-lengths, on the derived temperatures, we may assert that the true radiation of the solar surface may be represented by the known laws for purely thermal radiation of

perfectly black bodies of definite temperature. There are other topics which concern themselves with the same thing. In certain theories there is assumed a continuous transition of conditions in the sun as we pass from inside outward, and the observed limitation of the solar body is regarded as an optical appearance. The lack of validity of this assumption has been shown from various sides.[1] On the assumption of a continuous transition of condition in the sun, the distribution of brightness on the sun's surface has been derived by Schwarzschild[2] from an assumed equilibrium of radiation in the solar atmosphere. Independently of the fact that the equilibrium of radiation leads to an incomprehensible atmosphere which reaches into infinity with a very high constant temperature at its extreme strata, the distribution of the brightness of the white light on the solar surface derived from it gives deviations from those actually observed, which are appreciably larger than those resulting from our assumption. The distribution of intensity similarly derived by E. Oepik[3] for different wave-lengths also gives greater residuals than does ours. The solar temperature was also derived by D. A. Goldhammer[4] from the intensities of radiation in its spectrum, but without taking into account the absorption in the solar atmosphere, whence no agreement of the separate values could be expected. G. A. Shook[5] determined the solar temperature from the intensities of radiation directly observed for several wave-lengths at different points of the solar disk; and as was to be expected, the separate values vary in a manner corresponding to the progression of absorption for different wave-lengths on the sun's disk. C. G. Abbot[6] remarks that the intensities for different wave-lengths of the radiation coming to us from the whole visible solar surface, outside the earth's atmosphere, do not correspond to the distribution of energy in the spectrum of a perfect radiator of a definite high

[1] R. Emden, *Gaskugeln*, "Schmidt'sche Theorie."

[2] "Über das Gleichgewicht der Sonnenatmosphäre," *Nachrichten von der kgl. Ges. der Wiss. zu Göttingen*, **1**, 41, 1906.

[3] "Zur Theorie der Sonnenstrahlung," *Astronomische Nachrichten*, **198**, 49, 1914.

[4] "Die Temperatur der Sonne," *Annalen der Physik*, **25**, 905, 1908.

[5] "A Determination of the Sun's Temperature," *Astrophysical Journal*, **39**, 277, 1914.

[6] C. G. Abbot, *op. cit.*, 198–201.

temperature. If these total intensities are not reduced to those at the center of the sun's disk, and then subsequently corrected for the absorption in the solar atmosphere, we have no reason to expect an agreement of the corresponding curves.

II. OBSERVATIONS WITH FILTERS

In this part of the paper we shall examine my own observations of the radiation of the sun and its variation. The observations were made with filters in connection with the compensation pyrheliometer of Knut Ångström.

1. *The filter for radiation.*—I employed as monochromatic filters colored glasses from Jena, which transmitted, respectively, only limited regions in the red, green, and violet. The polished surfaces were 3×3 cm, and the thicknesses were: red, 4.898; green, 4.958; violet, 4.954 mm. The coefficients of transmission of the filters, i.e., the intensities of the transmitted radiation expressed in percentage of the incident radiation, were determined for different wave-lengths as follows. The radiation, in the one case of the sunlight, and in the other of a black body (that of O. Lummer and F. Kurlbaum) raised to a temperature of 1573° Abs., was dispersed in a spectrometer, and the intensities were then measured, both with and without the filters, by the galvanometer deflections from a linear thermopile. In the mean from such experiments, the values of the coefficients of transmission for the separate filters shown in Table VII were obtained.

TABLE VII

λ...	0.405	0.433	0.456	0.481	0.501	0.534	0.604	0.670	0.699	0.866	1.031	1.225	1.655	2.097
r...	0	1	2	4	3	3	9	86	87	91	88	84	87	86
g...	0	0	1	2	5	8	2	0	0	0	0	2	7	17
v...	10	15	12	6	3	1	0	0	0	0	0	0	1	1

2. *Determination of the solar constant, both complete and monochromatic, by means of the compensation pyrheliometer, with filters.*—The observations in question were made by me in part in the year 1913, at Sardar-Bulag, longitude $44^\circ 23'$, latitude $39^\circ 42'$, altitude 2350 m, in the Pass between the large and small Ararat in the Caucasus; but principally from the year 1912 at the University of

Warsaw. The method of observation with the pyrheliometer is well known.[1] The filters were placed in a cylindrical cap of the instrument. Table VIII represents a sample page of the records.

TABLE VIII

SATURDAY, JULY 12, 1913

Therm. = 17°.0–20°.0. Barom. 578.9–579.7 mm. Abs. humidity = 9.6 mm. Rel. humidity = 55 per cent

o without Filter		*r*		*g*		*v*	
Time	Angle	Time	Angle	Time	Angle	Time	Angle
5^h36^m	44°	5^h46^m	38°	5^h58^m	80°	6^h15^m	66°
	41		35		70		78
	45		38		82		67
	44		37		78		55
5 42	43	5 50	37	6 6	79	6 45	95
6 54	58	7 5	43	7 14	83	7 30	72
	54		42		95		75
	55		44		86		82
	54		43		105		78
6 58	58	7 8	44	7 26	61	7 42	73
7 46	61	7 52	46	8 5	86	8 18	85
	57		44		95		79
	64		44		93		84
	59		45		94		83
7 49	58	7 58	44	8 14	92	8 32	78
8 40	65	8 46	46	8 54	85	9 11	80
	62		45		95		75
	64		46		95		85
	59		44		95		80
8 43	64	8 49	46	9 5	95	9 23	87
10 44	64	9 37	46	9 51	88	10 7	93
	58		44		107		90
	68		49		92		85
	60		46		105		78
10 52	65	9 43	45	10 3	102	10 19	88
10 56	65	10 31	46	11 7	95	11 30	82
	61		48		90		85
	61		45		118		92
	61		46		91		63
11 00	67	10 37	47	11 22	85	11 44	120

The observations were usually made as follows: first, without filter; then with filter—red, green, violet, successively; and in each case the heating current was commuted five times, and the time at

[1] K. Ångström, "Kompensationspyrheliometer," *Wied. Ann.*, **67**, 633, 1899.

the beginning and end of each group was noted. The theories of extinction are strictly applicable for monochromatic light. They should theoretically yield for the earth's atmosphere larger coefficients of transmission in proportion as a greater path is covered, when white light is used,[1] but quantitatively, by experiment from photometric observations of stars, no such difference has been noted;[2] therefore, in what follows Laplace's extinction theory is employed, not only for the monochromatic observations, with filters, but also without them. According to this theory, if I_z represents the observed intensity of radiation of the sun at a zenith distance z, and if I denotes the same outside the earth's atmosphere, ξ the refraction, and p the coefficient of transmission of the latter, then we shall have

$$\log \frac{I_z}{I} = -K \frac{\xi}{\sin z} \qquad (1)$$

and

$$\log \frac{I_0}{I} = \log p = -K a_0 .$$

Since the refraction at $z=45°$ is represented by the series

$$\xi_{45} = a_0(1 - 0.001 \; . \; . \; . \; .) ,$$

we may employ, in the determination of p, the refraction at 45° for that incident in place of a_0. Further, there exists between the intensity of radiation sought, I_z, and the intensity i_z actually measured, the relationship

$$I_z = c \cdot i_z^2$$

or

$$I = c \cdot i^2 ,$$

where c is the constant of the pyrheliometer. Thus we get

$$\log i_z^2 = \log i^2 - K \cdot \frac{\xi}{\sin z} ,$$

or

$$Y = Y_1 - \tan \alpha \cdot X \qquad (2)$$

that is, the equation of a straight line. Therefore, if we draw from the abscissas $X = \frac{\xi}{\sin z}$ and ordinates $Y = \log i_z^2$, according to the

[1] C. G. Abbot, *op. cit.*, 2, 13. [2] G. Müller, *Die Photometrie der Gestirne*, p. 144.

observations with one filter, the corresponding points ought to lie in a straight line, the intersection of which with the Y-axis furnishes us with $Y_1=\log i^2$, and its acute angle with the X-axis, moreover, furnishes $\tan \alpha=K$.

The abscissas were determined by computing for the middle of the time of observation for each group the apparent zenith distance of the sun, and the true refractions from the meteorological data. The ordinates were determined by taking the mean of each five observed angles and multiplying it for the observations zero and r by 0.005A, and for g and v by 0.0005A, which furnishes us the intensities of the current. As the constant of the instrument the value $c=13.72$ was taken. The most probable values for $\log i^2$ and K are computed by the method of least squares from equations of condition of the following form,

$$-\log i_z^2+\left(\log i_0^2-K_0\frac{\xi}{\sin z}\right)+\delta(\log i_0^2)-\frac{\xi}{\sin z}\delta(K_0)=0\ ,$$

where the initial values were obtained from a straight line drawn approximately.

The results of the observations thus reduced are given in Tables IX A and IX B where the following abbreviations were used: t=the time; S_0=the full solar constant; S_r, S_g, and S_v, respectively, the portions transmitted by the filter; ϵ=its mean error in per cent; p=the coefficients of transmission; n=the number of groups of observations; T=the mean temperature; B=the barometric pressure; e=the absolute humidity; f=the relative humidity; and r=the "relative number" of sun-spots.

3. *Investigations of the short-period variations of the solar constant and their causes.*—In the first place, attention should be called to the effect noted in our observations of the outbreak of the volcano Katmai in Alaska on the general transparency of the earth's atmosphere; this was reported from many places, in the second half of 1912, during which period our observations began. Thus the coefficients of transmission p for white and colored light are especially small in Table IX B for August 1912 and the deviations of the solar constant S_0 are striking.

TABLE IX A—SARDAR-BULAG

t	S_o	ϵ	p	n	S_r	ϵ	p	n	S_g	ϵ	p	n	S_v	ϵ	p	n	T	B	e	f	r
1913 July 11	1.644	3	0.863	6	0.820	2	0.912	6	0.039	8	0.872	7	0.033	6	0.733	7	18.2	577.5	7.7	36	10
12	1.742	3	0.842	6	0.851	0	0.896	6	0.038	5	0.870	6	0.032	6	0.830	6	18.5	579.3	9.6	55	13
13	1.535	1	0.886	5	0.811	1	0.909	5	0.037	5	0.877	4	0.029	10	0.802	3	17.6	578.0	10.9	62	8
14	1.563	2	0.890	7	0.830	1	0.900	7	0.037	5	0.864	7	0.041	10	0.672	7	19.6	577.5	10.2	43	7

TABLE IX B—WARSAW

t	S_o	ϵ	p	n	S_r	ϵ	p	n	S_g	ϵ	p	n	S_v	ϵ	p	n	T	B	e	f	r
1912 Aug. 5	1.354	15	0.601	6	0.790	5	0.638	6	0.032	19	0.719	6	......	...	......	...	21.5	749.8	11.6	65	0
13	1.461	7	0.512	16	0.935	5	0.524	15	......	...	......	...	......	...	......	...	14.8	745.9	7.5	63	0
31	1.415	5	0.770	7	......	...	......	...	0.034	4	0.809	6	0.038	34	0.488	6	18.0	749.0	9.7	67	0
Sept. 14	1.366	11	0.707	7	......	...	......	...	0.031	4	0.795	7	......	...	......	...	11.2	753.5	7.2	77	19
Dec. 22	1.820	12	0.728	9	0.780	9	0.813	9	......	...	......	...	......	...	......	...	4.0	751.8	4.3	79	7
1913 Jan 4	1.549	12	0.784	5	1.096	11	0.786	5	0.036	17	0.838	5	......	...	......	...	− 0.7	764.6	4.3	99	7
16	1.384	6	0.837	5	0.847	1	0.858	5	0.029	10	0.888	4	......	...	......	...	− 6.3	757.3	1.6	56	7
1914 Apr. 29	1.528	5	0.812	7	0.820	4	0.851	7	0.036	6	0.833	4	......	...	......	...	16.5	752.4	6.8	49	(60)
May 4	1.750	3	0.793	8	1.069	2	0.770	7	......	...	......	...	......	...	......	...	11.0	758.8	4.0	44	(0)
15	1.603	3	0.789	5	0.982	1	0.795	4	......	...	......	...	......	...	......	...	11.3	758.8	5.2	55	(0)
19	1.734	3	0.803	7	0.807	2	0.912	7	......	...	......	...	......	...	......	...	18.2	754.5	7.1	48	(0)
23	2.023	5	0.655	8	1.114	2	0.691	7	0.063	5	0.711	4	......	...	......	...	16.9	759.8	7.0	52	(20)
25	1.535	4	0.780	10	0.895	5	0.780	10	......	...	......	...	......	...	......	...	20.5	749.7	8.2	50	(0)
June 13	1.816	5	0.765	8	0.817	4	0.886	8	......	...	......	...	......	...	......	...	20.8	752.9	7.2	42	(45)
14	1.888	3	0.753	7	0.757	4	0.941	7	0.045	3	0.671	6	0.032	4	0.690	6	20.6	752.0	7.2	42	(55)
15	1.549	5	0.827	6	0.914	3	0.788	6	0.041	5	0.749	5	0.040	5	0.600	5	21.9	751.8	9.8	53	(32)
July 2	2.070	5	0.607	5	0.942	7	0.789	5	......	...	......	...	......	...	......	...	21.8	754.0	11.1	58	(0)
23	1.507	2	0.789	8	0.968	3	0.723	8	......	...	......	...	......	...	......	...	26.0	740.1	14.1	59	(0)

But independently of this, there are noted further conspicuous oscillations of the values for the solar constant which follow each other at short intervals. On closer investigation, we may establish therefrom the fact that these oscillations of the solar constant stand in a relation with the variations of the corresponding coefficients of transmission of the earth's atmosphere: large values of the solar constant correspond to small values for the coefficients of transmission and vice versa.

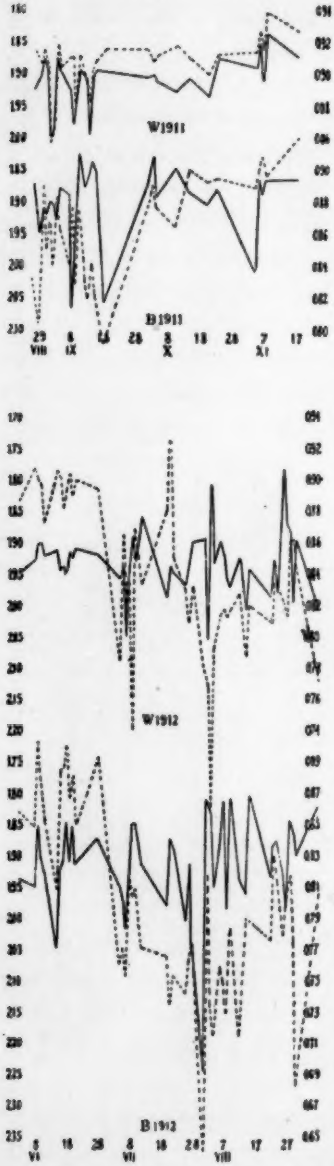
FIG. 2

To investigate this in a more general manner the corresponding results for white light for other observers are compared. The same relation came out in case of the observers between 1902 and 1907 at Washington; also between 1905 and 1912 at Mount Wilson, California, and from 1911 to 1912 at Bassour in Algeria.[1]

Fig. 2 represents graphically the observations which were made for the purpose of clearing up the nature of the short-period variations of the solar constant, which were obtained simultaneously in 1911 and 1912 at Mount Wilson (W), and in Bassour (B). The abscissas here denote the time, and the ordinates at the left denote the final values of the solar constant and at the right the apparent coefficients of transmission; here the solar constant is denoted by the solid line, and the coefficients of transmis-

[1] C. G. Abbot, *op. cit.*, 2, 96–98; 3, 102–24.

sion by the dotted lines. Since the ordinates increase at the left and right in the opposite sense, the result is that the two phenomena are certainly demonstrated to have a parallel progression.

We may furthermore note that the above-mentioned effect of the eruption of Katmai in the latter half of 1912 showed itself both at Mount Wilson and at Bassour in a simultaneous decrease of the coefficients of transmission at the end of June and still more at the end of July. But since a parallel progression of the coefficients of transmission with the solar constants is demonstrable at both stations, we may indirectly expect an approximately simultaneous occurrence of the extreme values for the solar constant at both stations. C. G. Abbot called attention to such close coincidence of the extreme values of the solar constant from these last observations at two widely separated stations; but he interpreted this as evidence for an actual, short-period variation of the sun's own radiation.[1]

Our observations, however, lead to the opinion that the cause of the variations is to be sought, not in cosmical, but in terrestrial meteorological phenomena and in the methods of reduction. It is interesting to make a comparison of the observations on two days with extreme values of the coefficients of transmission and of the solar constant yielded therefrom, as well as of the absolute humidities observed, because we thus get an indication of the effect of the transparency of the layers of our atmosphere on the phenomena in question.[2] The suspicion that the contrast between the portions

TABLE X

t	S	p	e
1907 February 15.............	1.872	0.797	1.45
May 14.............	2.034	0.660	14.60

of the sun at the center and the portions near the limb stands in some relation to the solar constant was therefore also brought into the question of such a connection with the meteorological phenomena.[3]

[1] *Ibid.*, 3, 2, 14, 117, 128, 168.

[2] *Ibid.*, 2, 98; 3, 101, 135.

[3] *Ibid.*, 3, 163–165.

We now have to consider how to interpret the fact that small values for the coefficients of transmission of the terrestrial atmosphere yield large values of the solar constant and vice versa. According to formulae (1) and (2) there will correspond to the small values of the coefficients of transmission p large angles of inclination a with the X-axis for the straight line representing the observations; similarly, large ordinates at the intersection of this line with the Y-axis correspond to large values of the solar constant. According to these equations we also have

$$I_z = I \cdot p^{\frac{X}{a_0}},$$

so that $\frac{X}{a_0} = F(z)$ denotes the path traversed by the light in units of the length of path for the zenith distance zero. Now we believe that on the days of feebly transparent atmosphere which have yielded small values for the coefficients of transmission, the strongly absorbing layers of the atmosphere lie relatively deep, at the surface of the earth, so that we may take for the paths traversed $F(z) = \sec z$. This would, however, yield larger abscissas X, particularly at large zenith distances, and in contrast with the values of the mean refraction as we have computed them. The consequence of this would be that the diminished angle of inclination a also depresses the crossing point of the Y-axis so that the value of the solar constant would be diminished. Consider similarly the days of very transparent atmosphere, with only feeble absorption in strata at high levels, when observations long before or after noon yield almost the same large value of radiation as at noon. Then the paths $F(z)$, increasing less rapidly with the zenith distance than those computed with the refraction, would furnish shorter abscissas X and consequently larger values for the solar constant. It therefore appears that it would be better in reducing the observations to introduce an expression of the form

$$I_z = I \cdot p^{F(z)^n} \tag{3}$$

where $n \gtreqless 1$ makes possible a larger variability for the length of path traversed. This can also be obtained if the lengths of path are determined in units of the varying height of the absorbing stra-

tum of the atmosphere. Let the height of the absorbing atmosphere, in units of the radius of the earth, be h, then we get for the zenith distance z from the triangle: observing station-earth's center-limiting point of the atmosphere, for the angle at the latter point

$$\sin \alpha = \frac{\sin z}{1+h};$$

hence for the angle at the center of the earth

$$\beta = z - \alpha$$

and therefore for the path in the atmosphere, in units of the height of the atmosphere h,

$$F(z) = \frac{\sin \beta}{h \sin \alpha}. \tag{4}$$

It is pleasing to find that the expressions (3) and (4) yield a better representation of the observations, even on a single day.[1] It would therefore be possible to solve the observations of a few successive days so that they can furnish the same most probable value of the solar constant I at different values of the unknowns p, n, or h; or to derive the most probable values of I from combining in a mean the results for a few successive days.

Inasmuch as the observations at elevated stations give smaller variations of the final values, we may conclude that the mean values employed in the first part of this paper, derived from very many observations of the intensities of the solar radiation as well as for the solar constant, are not affected by appreciable errors.

III. CONCLUSION

The very detailed and valuable spectro-bolometric investigations of the distribution of energy in the solar spectrum demand numerous corrections for the reduction of their final values and also require a very expensive arrangement of the observatory for making the observations. For investigating the variations over short intervals of time of the sun's own radiation in different colors,

[1] A. Bemporad, *Meteorologische Zeitschrift*, **24**, 306, 1907; and author's paper, *Astronomische Nachrichten*, **183**, 241, 1910.

the observations with the quasi-monochromatic filters and the corresponding reductions are sufficiently exact, and at the same time are very convenient and simple.

It is also possible to determine the temperature of the sun by the same process of observation with the filters, and this either for the effective temperature outside the earth's atmosphere or for the absolute temperature of the solar photosphere. This can be done by the method of isochromatic curves.[1] If the radiation of a black body at different temperatures is measured through the same filter which was used for the solar radiation, then the logarithms of the intensities of radiation, corrected to the same apparent size of the radiating objects, taken as ordinates, with the corresponding reciprocals of the absolute temperatures as abscissas, give a set of points which lie on a straight line. For the determination of the effective solar temperature, we take the part of the solar constant transmitted by the filter as the sun's intensity of radiation. But if we also measure[2] the intensities of the radiation through the same filter at different points of the sun's disk, and determine by the process given in I, 2 (this paper) the intensity of the radiation of the solar photosphere transmitted by the filter, we may derive from this same isochromatic curve the absolute temperature of the photosphere. Preliminary calculations necessary for this, as well as the preparatory experiments with the black body, have been carried out and are described in my original paper (*loc. cit.*).

On account of the great theoretical and practical importance of the investigations as to the possible variations in the solar radiation, either of short or long periods, it would be very desirable that observations of the sun similar to those described here with filters should be carried on simultaneously at numerous, widely separated stations over a long period of time.

WARSAW, PHYSICAL LABORATORY OF THE
IMPERIAL UNIVERSITY
May 1, 1915

[1] G. K. Burgess and H. la Chatelier, *The Measurement of High Temperatures*, 1913, pp. 321, 323.

[2] G. A. Shook, *op. cit.*

ON THE CHANGES IN THE SPECTRUM, PERIOD, AND LIGHT-CURVE OF THE CEPHEID VARIABLE RR LYRAE[1]

By HARLOW SHAPLEY

I. INTRODUCTION

The seventh-magnitude star RR Lyrae is the brightest representative of the cluster-type subdivision of the Cepheid variables.[2] Because of its brightness and its period of 13.5 hours the star has appropriately received more consideration of various kinds than other members of the class. Its study, therefore, may contribute more than that of the ordinary cluster-type star to our knowledge of the nature of Cepheid variation.

The variability of the light was first noted at the Harvard College Observatory by Mrs. Fleming prior to July 1899, and was announced in 1901.[3] The discovery was effected in a unique manner. An exposure of $6\frac{1}{2}$ hours on the field of the variable was interrupted for a few seconds every half-hour by means of an automatic device that stopped the driving clock and thus permitted the stars to drift to new places on the plate. The resulting rows of thirteen images furnished for each star a means of detecting any conspicuous short-period variation.[4] The new variable was immediately placed

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 112.

[2] We must here except β Cephei, for which the amplitude of light-variation is too small for ordinary photometric investigation. It is very likely that in the course of time many other naked-eye stars will be included among the cluster-type variables of small light-variation; for instance, β Canis Majoris for which the spectrum and the spectroscopic orbit are nearly identical with those of β Cephei (*Lick Observatory Bulletins*, **6**, 22, 1910), and 12 Lacertae (*Journal of the Royal Astronomical Society of Canada*, **9**, 423, 1915). Similarly other bright stars may be found to be Cepheids of longer periods and of small variation, such as is suggested very recently by Guthnick and Prager for α Cygni (*Astronomische Nachrichten*, **201**, 443, 1915). For 12 Lacertae, observations of the spectrum by the writer, using the 10-inch portrait lens and objective prism, have shown no conspicuous change throughout the four-hour period either in stellar magnitude or in spectrum lines.

[3] *Harvard Circulars*, No. 54, 1901; *Astronomische Nachrichten*, **154**, 423, 1901.

[4] *Harvard Circulars*, No. 29, 1898; *Astronomische Nachrichten*, **147**, 93, 1898.

upon the observing-list of the polarizing photometer at Harvard, and the long series of measures by Professor Wendell are of highest value in studies of the star's peculiarities.[1]

That RR Lyrae is a spectroscopic variable of the typical Cepheid variety was discovered at the Lick Observatory by Kiess, who also published photometric observations of the star and some considerations of changes in the intensity of the continuous spectrum.[2] As is usual for Cepheid variables the spectroscopic study discloses an abnormally small mass function and a very small apparent orbit, while no trace of a secondary spectrum is found. The periodic shift of the spectral lines suggests the explanation that the variable moves in an orbit about an obscure companion, but the improbability of the double-star interpretation has been pointed out in a previous article.[3] It was found, in fact, from Wendell's photometric observations of RR Lyrae that the time of the rise to maximum light varied greatly from that required by a uniform period,[4] and that to a certain extent the variation was erratic. More recently Martin and Plummer, in a photographic study of the light-curve, have considered the possible interpretation of this type of variation, concluding that the evidence is strongly against the double-star interpretation, and that a hypothesis involving pulsations is much more reasonable.[5]

The present communication contains a further discussion of the anomalies of the light-variation, as far as they are shown in the several available series of measures, as well as a report on some observations of the star's spectrum. It is hoped that through investigations of this kind the true explanation of Cepheid variation can be definitely determined, and this in turn should throw light on the method of stellar evolution.

[1] *Harvard Annals*, **69**, Part I, 45, 1909; Part II, 124, 1914.

[2] *Lick Observatory Bulletins*, **7**, 140, 1913.

[3] "On the Nature and Cause of Cepheid Variation," *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[4] *Publications of the American Astronomical Society*, Report of the 16th Meeting, p. 16, 1914; *Popular Astronomy*, **22**, 144, 1914.

[5] *Monthly Notices*, **75**, 566, 1915.

II. VARIATION OF THE SPECTRUM

Harvard classifies the spectrum of RR Lyrae as F.[1] Kiess observes that the hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ are of equal intensity and that the calcium line K is about 0.8 as intense as $H+H\epsilon$.[2] On a photograph of the spectrum, made by the writer with a 15-degree prism mounted in connection with the 10-inch Cooke photographic triplet, the line K was seen to be less than 0.1 the intensity of $H+H\epsilon$. A series of observations was accordingly undertaken for the purpose of determining the class of spectrum at different phases of the light-variation. The results are collected in Table I.

Most of the plates listed in the first column contain multiple images, and the corresponding exposures are designated by post-script letters. The phases in the fifth column will be discussed in a later section of the paper. The brightness in the seventh column increases with the number designating it.

TABLE I

THE SPECTRUM OF RR LYRAE

No. Plate	Date	G.M.T.	Length of Exposure	Phase	Spectrum	Brightness	Remarks
	1915						
28*a*......	Nov. 27	$16^h 55$	10^m	-0^d031	A0	5	
28*b*......	....	17 22	15	− .013	A0	5	Low
32*a*......	Dec. 8	14 22	10	+ .093	A6	2	
32*b*......	....	14 33	10	+ .100	A8	2	
33.......	....	15 23	15	+ .135	A5	2	
34*a*......	....	16 9	11	+ .167	A8	1	Diffuse
34*b*......	....	16 23	11	+ .177	A8	1	
42*a*......	Dec. 9	14 51	10	− .021	B9	5	
42*b*......	....	15 4	10	− .012	A0	4	
42*c*......	....	15 19	15	− .001	A2	4	
44.......	....	17 20	10	+ .083	A4:	3	Haze
52*a*......	Dec. 10	14 29	16	+ .397	A8:	0	Faint
52*b*......	....	14 49	24	+ .411	F2	0	
53.......	....	16 8	16	+ .466	A7:	1	Clouds
55.......	....	17 16	11	+ .513	A2:	3	Drift
56*a*......	Dec. 11	14 45	10	+ .275	>A2	2	Clouds
56*b*......	....	15 4	26	+0.288	>A5	2	Clouds

A conspicuous change of spectrum with magnitude is at once apparent. The determinations of the class, except in the cases marked as uncertain, are fairly definite, though high accuracy is not

[1] *Harvard Annals*, **55**, 25, 1907; **56**, 194, 1912. [2] *Op. cit.*, p. 144.

to be expected because of the small dispersion. The classification was made without knowledge of the phase of variation or the relative brightness of the star, and similarly the estimate of the integrated brightness of the spectrum, relative to that of neighboring

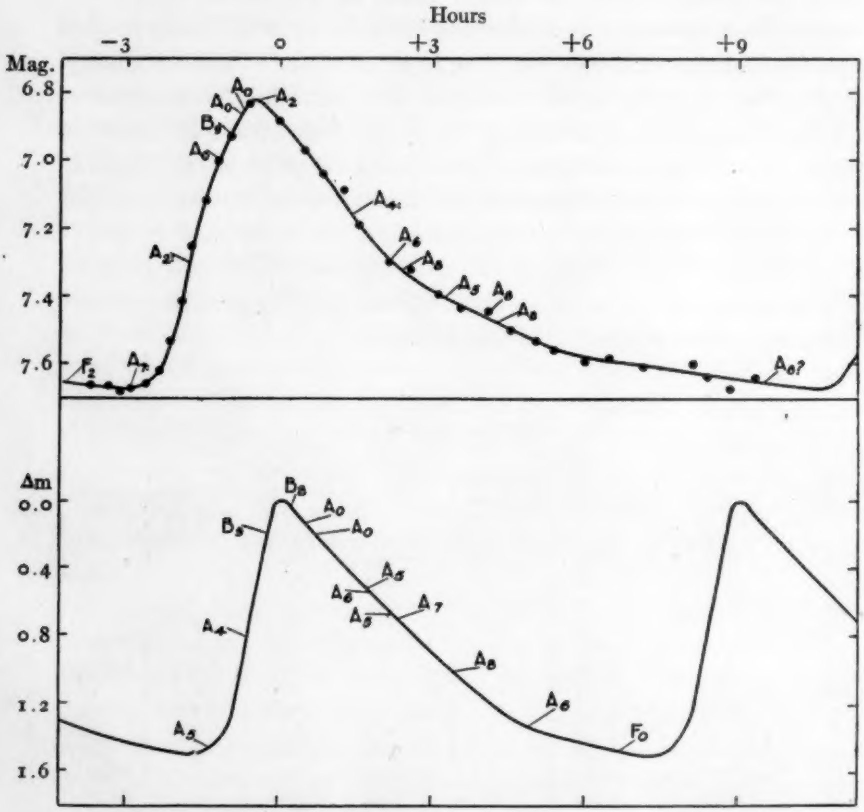

FIG. 1.—*Top:* Mean visual light-curve of RR Lyrae (observations by Wendell) and variations of spectrum (Shapley).

Bottom: Photographic light-curve of RS Boötis (Seares and Shapley) and variation of spectrum (Pease).

stars, was made without knowledge of the phase or the spectral type. The relation of the variation of spectrum to variation in light is shown in Fig. 1, the spectral classification being plotted along the mean light-curve derived from Wendell's observations.

A notable feature of this change of spectrum with changing light is that the variation is from one recognizable spectral type to another. Moreover, it is a change from one spectral type to another that adjoins it in the usual conception of the evolutionary sequence of spectral classes. We might say that in this star we have visible proof of the evolution of stars along a spectral series—a phenomenon that is universally believed to exist, but for which our evidence heretofore (if we except the phenomena of novae) is altogether the indirect evidence of the continuous gradations from one star to the next of various constant characters (such as spectrum, motion, intrinsic luminosity), and not the direct fact of variation.[1]

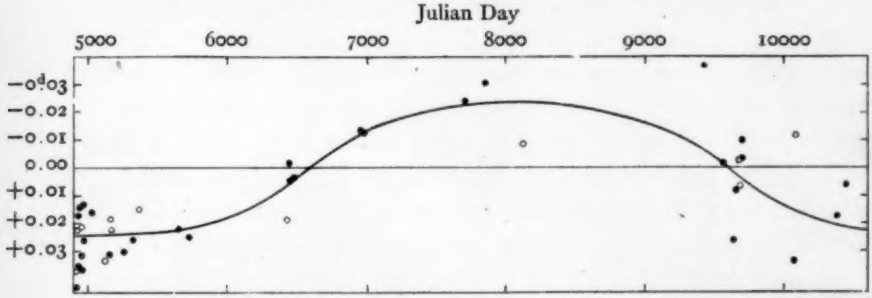

FIG. 2.—Long-period variation in light-elements of RR Lyrae. Open circles indicate observations of weight less than 3.

The data of Table I could be increased without trouble, but beyond establishing the qualitative result there is little more to be done to advantage with low dispersion. The change of spectrum with brightness is considered proved. This is in complete agreement with the results for SW Andromedae[2] and RS Boötis.[3] For all variables of the cluster type we should expect a similar variation of the spectrum, at least for those for which the photographic range exceeds the visual.

The photographic range of RR Lyrae, according to Martin and Plummer, does not exceed the visual, a result hardly to be expected

[1] See "A Short Period Cepheid with Variable Spectrum," *Proceedings of the National Academy of Sciences*, **2**, 132, March, 1916.

[2] *Mt. Wilson Contr.*, No. 92, p. 10; *Astrophysical Journal*, **40**, 457, 1914.

[3] *Mt. Wilson Contr.*, No. 92, p. 10; *Astrophysical Journal*, **40**, 457, 1914; *Publications of the Astronomical Society of the Pacific*, **26**, 256, 1914.

if the change in spectrum is from a normal A type at maximum to the redder F type at minimum.[1] Their result, however, is not necessarily conclusive, for none of the photographic series of measures completely covers the maximum. Moreover, Fig. 3 indicates that the individual maxima may vary greatly in height. It may be, then, that simultaneous observations of the spectrum and of the photographic and visual light-curves, referring the light-curves

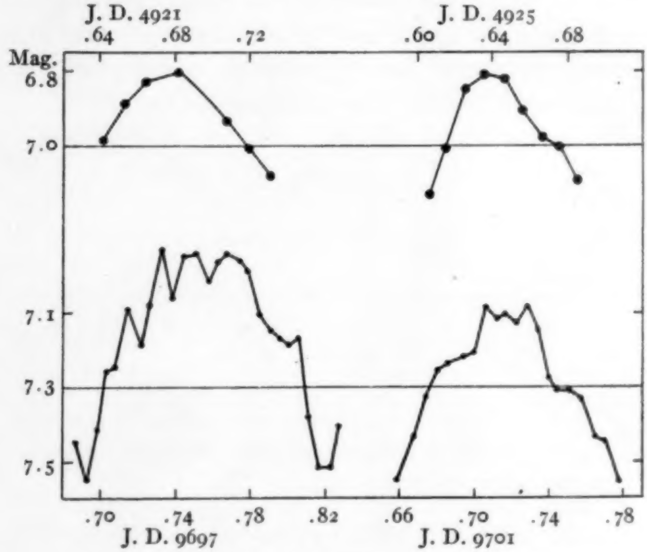

FIG. 3.—Variations in the shape of maxima of RR Lyrae.

to standard magnitude scales, would show either a visual range less than the photographic, or, when equal, no change in the spectrum.

III. THE MEAN PERIOD AND ITS SLOW VARIATION

From the first year's observation of RR Lyrae, Wendell found the approximate period $0^{d}.5668$, and chose as a convenient epoch of reference the round numbers J.D. 2414856.5000, G.M.T.[2] In this case, as in many others where light-elements have been determined at Harvard, the initial epoch is not intended to be coincident with a principal phase of the light-variation—maximum or mini-

[1] The variation in photographic magnitude is 0.80, and visually (Wendell) is 0.84.

[2] *Harvard Annals*, **69**, Part I, 95, 1907; Part II, 165, 1914.

mum.[1] In the present case the chosen epoch comes nearly an hour after the observed maximum, but the two have been used as coincident in various discussions of the period. This has led to some error in determinations of the light-elements and particularly in comparisons of photographic and visual variations; but the chief criticisms of the two or three recent discussions must be that the long and important series of Harvard observations have been almost completely neglected, and that the periodic irregularities in the form of the light-curve have been ignored.

No attempt to improve the light-elements was made at Harvard after 1900, and the phases of all the observations by Wendell are computed from the foregoing approximate elements. Hertzsprung found that the mean period should be lengthened,[2] deriving the value $0^{\mathrm{d}}.56682$. Fontana[3] suggested, on the basis of data he considered weak, that the period should be still longer, $0^{\mathrm{d}}.5668267$; and Kiess derived at the same time the value $0^{\mathrm{d}}.566826$, finding difficulty in harmonizing photographic and visual observations. In the most recent discussion Martin and Plummer conclude that the period is increasing secularly from the value given by Wendell in 1899 to $0^{\mathrm{d}}.566863$ in 1914, giving the formula for maximum:

$$\text{J.D. } 2414856.470+0.566798E+(10^{-4}E)^2\div 3. \qquad (1)$$

That the period of RR Lyrae is not constant is shown clearly in the two Harvard series.[4] Plotting the observations, it is also immediately evident that Wendell's approximate period soon fails to represent his measures. A better uniform period for the observations of the first three years is the same one that, within the known errors and irregularities, represents the observations by Martin twelve years later. The period, then, is probably not longer now than it was fifteen years ago, but in the interval it has not remained constant.

A closer examination of this question, which may have an important bearing in the interpretation of Cepheid variation, is afforded

[1] See a case cited in *Popular Astronomy*, **20**, 656, December 1912.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **46**, 284, 1911.

[3] Fontana was the first to suspect a variation in the mean period, *Memorie della Società degli Spettroscopisti Italiani*, Serie II, **2**, 188, 1913.

[4] We are not yet considering the short-period oscillations.

by the material collected in the following tables. The various sources of light-observations, so far as they are known to the writer, are enumerated in Table II.

In all these series of measures very few continuous sets cover light-maximum, and but little could be determined of the variations in the period if we depended only on actual measures at the brightest phase. On the other hand, a great many determinations are possible of some selected point on the steep rising branch of the curve, for much of Wendell's work was evidently planned with an investigation such as the present in view. His series of nightly measures usually cover only the steepest part of the ascending branch, and, though they serve but little in showing directly the changes in the shape of the curve, they are of prime importance in studying the period, and especially in demonstrating the oscillations in the time of the rise to maximum light.

TABLE II

SOURCES OF OBSERVATIONS OF RR LYRAE

Observer	Place	Publication	Observing Method	Interval	Number
Wendell I	Harvard	*H.A.*, **69**, 45	Polarizing	1899.6–1902.1	241
Wendell II	Harvard	*H.A.*, **69**, 124	Polarizing	1903.8–1907.7	61
Von Zeipel	Upsala	*A.N.*, **177**, 372	Zöllner	1906.9–1908.0	38
Haynes	Laws Obs.	Unpublished	Zöllner	1908.4–1908.5	24
Hertzsprung	Potsdam	Unpublished*	Photogr.	....	160
Fontana	Catania	*Spett. It.*, **2**, 183	Wedge	1912.4–1912.5	44
Townley	Lick Obs.	*L.O.B.*, **7**, 141	Wedge	1912.6–1912.7	40
Kiess	Lick Obs.	*L.O.B.*, **7**, 141	Wedge	1912.6–1912.9	130
Shapley	Princeton	This paper	Polarizing	1913.8–1913.9	28
Martin	Dunsink	*M.N.*, **75**, 566	Photogr.	1913.8–1914.9	108

Vierteljahrsschrift der Astronomischen Gesellschaft*, **49, 192, 1914. For Hertzsprung's notes on the proper motion, see *Astronomische Nachrichten*, **196**, 205, 208, 1913.

The time on the ascending branch when the magnitude is halfway between the minimum and maximum values has been chosen as the epoch of reference, and for convenience will be referred to as the time of *median magnitude*. The method of determining these times is as follows: Neglecting the computed and published phases, the observations from all sources are reduced to Greenwich heliocentric mean time and plotted on a uniform scale, furnishing fragmentary light-curves for each Julian Day on which the star was

observed.[1] A mean light-curve, plotted to the same scale on tracing paper, is superposed on the plot of each night's measures and adjusted in the time co-ordinate so as to satisfy all the observations as closely as possible. The time of the median magnitude is then read off, with an error of not more than two or three minutes in a good series of observations.

For the photographic observations the mean curve by Plummer and Martin was used. To determine the time of median magnitude for the visual observations a mean curve was derived from Wendell's

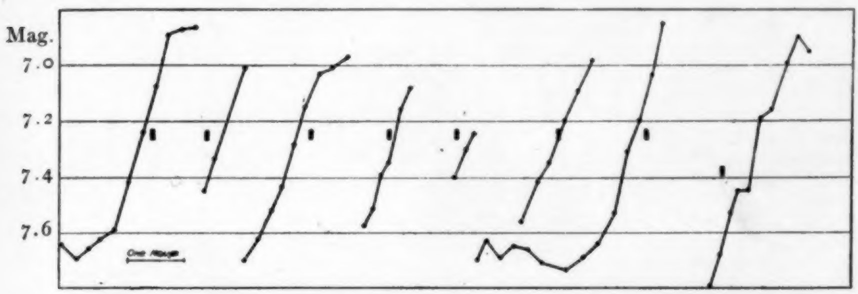

FIG. 4.—Observed ascending branches and computed times of median magnitude for epochs 144, 195, 225, 765, 2755, 3766, 5271, 9210. The existence of the short-period oscillation is illustrated.

first series. Allowance was made in each observer's work for the difference in range of variation, a factor depending on the choice of comparison stars as well as on the scale of the photometer. To form the mean visual light-curve the phases in *Harvard Annals*, **69**, Part I, are corrected to conform to the period $0^{d}.566826$, and the initial epoch adopted by Wendell is retained. The resulting normal magnitudes are given in Table III and the curve is plotted in Fig. 1.[2]

It should be noted that the determination of mean curves for stars of this type probably has no value beyond occasional use, such as the present. The great deviation in the form at different epochs[3]

[1] Such as are plotted in Fig. 4.

[2] It was found after the completion of this work that the correction for the equation of light applied at Harvard to the phases in this series of measures was made with the wrong sign. As the correction is not large, the error has little effect on the mean curve. The times of observation recorded at Harvard are geocentric.

[3] See Fig. 3.

makes a mean curve representative of only those possibly widely unlike maxima which chance to compose it, and at most but a rough model of the curve at an unobserved maximum.[1] With this idea in mind no attempt is made to derive a definitive mean curve from all the observations, or even from the homogeneous Harvard series alone.

TABLE III

NORMAL MAGNITUDE CURVE OF RR LYRAE

No.	Phase	No. Observations	Magnitude	No.	Phase	No. Observations	Magnitude
1......	0ᵈ018	8	6.97	18......	0ᵈ369	4	7.67
2......	.034	5	7.04	19......	.388	7	7.64
3......	.051	7	7.09	20......	.412	3	7.66
4......	.065	3	7.19	21......	.426	7	7.66
5......	.087	3	7.30	22......	.436	10	7.68
6......	.108	4	7.32	23......	.444	6	7.67
7......	.130	3	7.39	24......	.456	9	7.66
8......	.148	3	7.43	25......	.466	8	7.62
9......	.169	7	7.44	26......	.475	9	7.53
10......	.190	5	7.50	27......	.486	15	7.41
11......	.209	6	7.53	28......	.495	17	7.25
12......	.230	8	7.56	29......	.505	22	7.12
13......	.248	4	7.59	30......	.515	18	7.00
14......	.268	6	7.58	31......	.525	12	6.93
15......	.296	4	7.61	32......	.540	5	6.83
16......	.336	4	7.60	33......	0.564	5	6.88
17......	0.348	4	7.64				

The short series of observations by von Zeipel, Haynes, and Fontana contain no sets of successive observations sufficient to give the time of the rise to single maxima, but in each case a mean curve of more or less weight can be obtained which gives a determination of the time of median magnitude for the mean date. These observations cannot be used, however, to discuss short-period oscillations.

The observations by the writer, made with the polarizing photometer and 23-inch refractor of the Princeton Observatory, have been reported upon briefly in an earlier note.[2] The individual observations are now given in Table IV. The comparison star is

[1] Harmonic analyses of the light-variations of cluster-type stars, if they are to have any meaning, must make allowance for these wide oscillations. See, for example, the diagrams of maxima of XX Cygni, *Astrophysical Journal*, **42**, 159, 160, 1915.

[2] *Popular Astronomy*, **22**, 144, 1914.

B.D.+42°3328, for which the Harvard[1] visual magnitude is 8.96. My observations show the same range as Wendell's, but the whole curve is $0^{M}.15$ below his, evidently because the comparison stars are different and their adopted magnitudes may have errors of that amount. Only one measure of the time of the median magnitude is obtainable from this series, but that determination is of considerable weight (Fig. 4).

TABLE IV

PRINCETON OBSERVATIONS OF RR LYRAE

No.	E.S.T.	G.H.M.T.	Julian Day	Mag. Diff.	Mag.	Remarks
	1913		2420000+			
1	Oct. 23 $6^h 16^m$	$11^h 16^m$	64.469	1.15	7.81	
2	27.5	28	478	1.15	7.81	
3	38	38	485	1.17	7.79	
4	49	49	499	1.18	7.78	Clouds
5	7 4.5	12 5	503	1.26	7.70	Thick
6	15	15	510	1.24	7.72	
7	30	30	521	1.22	7.74	Clouds
8	45	45	531	1.23	7.73	
9	Nov. 5 5 56	10 56	77.456	1.24	7.72	
10	6 5.5	11 5	462	1.11	7.85	
11	16.5	16	469	1.10	7.86	
12	26	26	476	1.11	7.85	Haze
13	43	43	488	1.14	7.82	
14	54.5	54	496	1.28	7.68	
15	7 4.5	12 4	503	1.43	7.53	
16	13	13	509	1.51	7.45	
17	23.5	23	516	1.51	7.45	
18	35	35	524	1.77	7.19	
19	45	45	531	1.80	7.16	
20	8 0	13 0	542	1.97	6.99	
21	12	12	550	2.07	6.89	
22	23	23	558	2.01	6.95	
23	Nov. 17 5 35	10 34	89.440	1.98	6.98	
24	46	45	448	1.97	6.99	
25	57	56	456	1.96	7.00	
26	6 7.5	11 6	462	1.98	6.98	
27	20	19	472	1.90	7.06	
28	30	29	478	1.85	7.11	

The epoch of the one maximum (photographic) available from Hertzsprung's observations is taken from the statement by Kiess.[2] It is corrected to the time of the median magnitude by means of the Dunsink mean photographic curve—a doubtful procedure, hence the low weight assigned it in Table V.[3]

[1] *Harvard Annals*, **63**, 178, 1913.

[2] *Op. cit.*, p. 140.

[3] The observation is not plotted in Fig. 2.

The data for the discussion of the period are summarized in Table V. The heliocentric times in the second column refer to the median magnitude. The number of the epoch in the third column is counted from the initial epoch used by Wendell. Weights depend on the number and arrangement of the observations and on the general reliability of the photometric method involved.

TABLE V

OBSERVATIONS OF MEDIAN MAGNITUDE

No.	J.D. and G.H.M.T.	Epoch	Observer	Weight	$O-C_1$	$O-C_2$	$O-C_3$
	2400000+						
1.....	14913.690	100	Wendell I	2	$+0^d.015$	$+0^d.012$	$+0^d.014$
2.....	14921.624	114	"	7	+ .013	+ .010	+ .012
3.....	14925.599	121	"	6	+ .021	+ .018	+ .020
4.....	14933.514	135	"	2	.000	− .004	− .002
5.....	14938.611	144	"	9	− .005	− .009	− .007
6.....	14947.684	160	"	3	− .001	− .005	− .003
7.....	14967.538	195	"	4	+ .014	+ .010	+ .012
8.....	14968.667	197	"	4	+ .009	+ .005	+ .007
9.....	14980.554	218	"	9	− .007	− .011	− .009
10.....	14984.521	225	"	8	− .008	− .012	− .010
11.....	14993.602	241	"	6	+ .004	.000	+ .002
12.....	15018.533	285	"	8	− .006	− .010	− .008
13.....	15150.621	518	"	2	+ .011	+ .007	+ .009
14.....	15171.592	555	"	4	+ .009	+ .005	+ .007
15.....	15175.550	562	"	2	.000	− .004	− .002
16.....	15184.616	578	"	3	− .004	− .008	− .006
17.....	15290.625	765	"	7	+ .008	+ .004	+ .005
18.....	15315.561	809	"	7	+ .004	+ .001	+ .002
19.....	15387.538	936	"	3	− .007	− .010	− .009
20.....	15663.591	1423	"	5	.000	− .001	.000
21.....	15743.517	1564	"	6	+ .003	+ .002	+ .003
22.....	16418.608	2755	Wendell II	3	− .001	+ .012	+ .012
23.....	16443.528	2799	"	8	− .021	− .008	− .008
24.....	16456.571	2822	"	5	− .015	− .001	− .001
25.....	16477.543	2859	"	6	− .016	− .001	− .001
26.....	16991.641	3766	"	8	− .033	.000	− .001
27.....	17000.711	3782	"	9	− .032	+ .001	.000
28.....	17703.002	5021	Von Zeipel	9	− .043	− .001	− .003
29.....	17844.704	5271	Wendell II	10	− .049	− .006	− .008
30.....	18122.474	5761	Haynes	1	− .026	+ .018	+ .016
31.....	18919.393	7167	Hertzsprung	2	− .070	− .032	− .035
32.....	19570.165	8315	Fontana	9	− .018	+ .004	.000
33.....	19635.946	8431	Townley	7	+ .010	+ .029	+ .025
34.....	19659.735	8473	"	6	− .008	+ .010	+ .006
35.....	19692.600	8531	Kiess	2	− .019	− .003	− .007
36.....	19693.743	8533	"	3	− .009	+ .007	+ .003
37.....	19697.694	8540	"	6	− .026	− .010	− .014
38.....	19701.668	8547	"	6	− .020	− .004	− .008
39.....	20077.515	9210	Shapley	10	+ .019	+ .024	+ .019
40.....	20093.341	9238	Martin	2	− .027	− .022	− .027
41.....	20390.388	9762	"	6	+ .002	+ .002	− .003
42.....	20453.296	9873	"	4	−0.009	−0.009	−0.014

The residuals in the sixth column, $O-C_1$, show the representation of the observations by the linear formula:

$$\text{Med. Mag.} = \text{J.D. } 2414856.992 + 0^{d}566830\,E. \qquad (2)$$

The agreement is fair at the beginning and end of the series, but for an intervening interval of ten years all residuals are negative, and the probability of a long-period change is at once apparent. Plotting $O-C_1$ against the epochs, it is found by graphical methods that the following harmonic expression represents these residuals with considerable accuracy:

$$-0^{d}020 - 0^{d}024 \sin\,(0^{\circ}0340\,E - 104^{\circ}5). \qquad (3)$$

The period of this variation is 10600 E, or about 16.5 years, and the amplitude is more than an hour. The necessity of the correction is exhibited in Fig. 2; the available data are somewhat fragmentary, however, and it cannot be claimed that the formula will satisfy fully the future deviations from a uniform period. The recent rough estimates of magnitude, listed in Table I, were made a year later than any observations involved in the derivation of the revised light-elements, and they appear to be in very good agreement with prediction. The phases in that table are computed by means of formula (5) below.

The residuals in the seventh column of Table V, $O-C_2$, result from correcting those of the preceding column by means of (3). The new residuals were then discussed by least squares to determine corrections to the initial epoch, T, and the mean period, P. Forty-two equations of condition were involved in the solution, and the corrections in days are as follows:[1]

$$\left.\begin{aligned}\Delta T &= -0.0020 \pm 0.0011\\ \Delta P &= +0.0000074 \pm 0.0000218\end{aligned}\right\} \qquad (4)$$

Applying these corrections, reducing from the median magnitude to maximum light, and adding the correction (3), the heliocentric light-elements that represent all the observations to date are:

$$\begin{aligned}\text{Max.} = \text{J.D. } 2414856.451 + 0^{d}566831\,E\\ -0^{d}024 \sin\,(0^{\circ}0340\,E - 104^{\circ}5).\end{aligned} \qquad (5)$$

[1] The large probable error of the period is accounted for in Section V.

The representation by this formula is shown by the residuals in the last column of Table V and by the deviations from the sine curve in Fig. 2. They will be discussed further in Section V.

IV. CHANGES IN THE FORM OF THE LIGHT-CURVE

That a wide variation exists in the form of the light-curve at different maxima has been shown directly for a few cluster-type variables,[1] and the oscillations in the time of the median magnitude all but prove the same thing for others of this class.[2]

Fig. 3 contains diagrams of maxima of RR Lyrae which show that for it also the curve changes its shape very conspicuously and rapidly. The two upper curves are taken from Wendell's earliest observations. The maxima are only seven periods apart, but the interval of time when the star was brighter than $7^{M}.0$ is in one case $1^{h}55^{m}$, and in the other nearly 40 per cent less. The lower pair of curves in Fig. 3 are from the Lick Observatory measures by Kiess. The interval when the brightness exceeded $7^{M}.3$ is about twice as great at the first maximum as at the second, which follows by seven periods. Moreover, the brightness at maximum differs by $0^{M}.15$.[3] If the photographic observations by Martin chanced to refer to maxima of the "faint" kind, the apparent equality of photographic and visual ranges may have no real significance.[4]

It is obvious from the diagrams that the determination of the time of median magnitude involves directly the shape of the curve. It suggests that oscillations in that time indicate variations in shape rather than actual oscillations in the epoch of maximum light. Apparently the recovery from a delayed outburst is more rapid than from an early one; that is, the factors that cause the late rise to

[1] XX Cygni, *loc. cit.;* SW Draconis and SU Draconis, *Astronomische Nachrichten*, **184**, 241, 1910.

[2] ST Ophiuchi, SW Andromedae, RR Lyrae, *Mt. Wilson Contr.*, No. 92, p. 5; *Astrophysical Journal*, **40**, 452, 1914.

[3] If we should attribute the difference in maximum magnitude to a variation in the comparison star, the curves would still be conspicuously dissimilar.

[4] Similarly, perhaps, we should not accept finally the hump on the photographic curve three hours after maximum, for there remains the strong possibility that some of the measures near that phase refer to narrow maxima and others to the wider variety.

maximum light generally allow an early decline, and vice versa. These peaked and round-topped curves are shown more completely in the figures accompanying the earlier paper relative to XX Cygni.

V. OSCILLATIONS IN THE TIME OF THE RISE TO MAXIMUM LIGHT

From the least-squares solution for the light-elements the probable error of a unit-weight determination of the time of median magnitude is $\pm 0^{d}0167 = \pm 24$ minutes. For the average determination listed in Table V, therefore, the probable error is more than ten minutes and for none is it less than seven minutes. Such large uncertainties in the observations, however, are clearly impossible. In many of Wendell's measures of the rise to maximum light the error in determining the time of median magnitude cannot well exceed two minutes, and from an inspection of all the plotted curves on which Table V is based the probable error of unit weight would be estimated at less than ± 8 minutes. The curves given in Fig. 4 illustrate this point fully. All the plotted measures are from Wendell except the last, which is taken from the writer's observations. The short heavy vertical line near each series indicates the time of the median magnitude for that maximum, computed from the adopted light-elements (5).

There can be no doubt that a real irregularity is present. An attempt was made to find a uniform period for the variation that would satisfy all the observations. This failed in part, perhaps because of insufficient data, but it seems that for the whole series the oscillation is roughly periodic with a varying amplitude. For the first 14 months of Wendell's measures, however, the variation is definitely periodic. Throughout that interval the following simple formula represents the residuals, $O-C_3$, with considerable accuracy:

$$(O-C_3)-0^{d}004-0^{d}013 \cos \frac{2\pi}{70} (E-45)=0. \qquad (6)$$

Table VI contains the data relative to this correction. The residuals from (6) are given in the fifth column, and the last column contains the difference between the weighted squared residuals before and after correcting. The existence of the periodic variation

is completely established. The sum of the squares of the residuals is decreased from 0.007762 to 0.001034, which gives, in this inter-

TABLE VI

THE APPLICATION OF FORMULA (6)

No.	Epoch	Weight	O−C$_3$	O−C$_4$	$(pvv)_3-(pvv)_4$
1	100	2	$+0^d.014$	$+0^d.007$	+0.000294
2	114	7	+ .012	− .005	+ .000833
3	121	6	+ .020	+ .005	+ .002250
4	135	2	− .002	− .003	− .000010
5	144	9	− .007	.000	+ .000441
6	160	3	− .003	+ .001	+ .000024
7	195	4	+ .012	.000	+ .000576
8	197	4	+ .007	− .003	+ .000160
9	218	9	− .009	.000	+ .000729
10	225	8	− .010	− .002	+ .000768
11	241	6	+ .002	− .006	− .000192
12	285	8	− .008	.000	+ .000512
13	518	2	+ .009	+ .004	+ .000130
14	555	4	+ .007	+ .006	+ .000052
15	562	2	− .002	+ .003	− .000010
16	578	3	− .006	.000	+ .000108
17	765	7	+0.005	+0.004	+0.000063

val, for the new probable error of an observation of weight unity ±9 minutes. The variation is shown diagrammatically in Fig. 5.

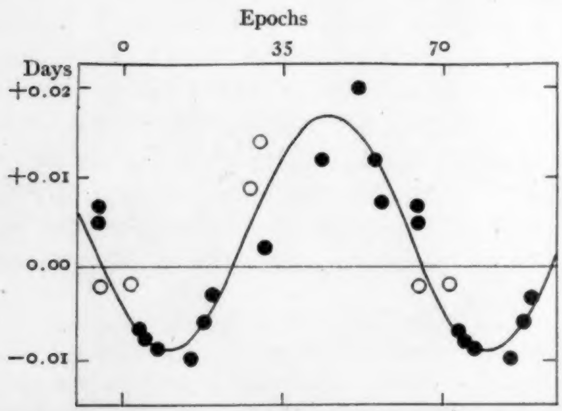

FIG. 5.—Short-period variation in time of median magnitude.

Eleven oscillations are involved; the period is 70 epochs or 40 days, and the amplitude is 37 minutes.

If we make the reasonable assumption that the errors of all the observations could be reduced proportionately as much as these earlier ones by analogous corrections for short-period irregularities in the median magnitude, then the probable errors from the least-squares solutions for T and P, given by (4), would be reduced from $\pm 0\overset{d}{.}0011$ and $\pm 0\overset{d}{.}000022$ to $\pm 0\overset{d}{.}0004$ and $\pm 0\overset{d}{.}000008$, respectively.

VI. SUMMARY

The present paper discusses in some detail the peculiarities of the light-variations of the Cepheid variable RR Lyrae, and considers the variations of its spectrum as observed at Mount Wilson. The star is the brightest of the sub-group with periods of a half-day. It is shown in Section II to be a variable in spectrum as well as in light and velocity. The computations of the third section demonstrate upon the basis of the work of nine observers throughout an interval of fifteen years that the mean period is affected by a slow variation which may complete its cycle in 16.5 years. The fourth section contains diagrams to show that the maxima of the light-curve are not constant in form, and in the fifth the closely analogous phenomenon of variations in the time of the rise to maximum light is proved to exist.

Mount Wilson Solar Observatory
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NOTES ON CERTAIN ULTRA-VIOLET SPECTRA

By F. A. SAUNDERS

Information about many spectra in the region near λ 2000 is scanty, or lacking altogether. Here both quartz and the air begin to absorb, and the reflection of speculum metal falls off, while the gelatine of ordinary photographic plates offers a further obstacle to successful investigation. It is the purpose of this paper to present a few observations in this part of the spectrum, which have been made at various times and with various experimental arrangements.

All the results here given were obtained by photographs taken on Schumann plates, using a one-meter concave grating, purchased through the generosity of the Committee of the Rumford Fund. This grating was mounted, either in air in an ordinary way, or inclosed in a brass tube a little more than a meter long, and 10 cm in diameter, which could be exhausted. In this latter arrangement, the slit and the plate were placed just outside the tube, at one end, the tube being sealed with suitable quartz plates.

In case the spark was used as a source of light, it was placed as close as possible to the slit. The light had then to travel a distance of about a millimeter through the air to the first quartz plate. This plate was 1.5 mm thick. The light then went down the one-meter tube, and on its return from the grating it passed through about 5 mm more of quartz, and a few tenths of a millimeter more of air, to the plate. The tube was kept at a cathode-ray vacuum, as shown by a small discharge tube, sealed in by a side connection, and was washed out repeatedly with hydrogen before exhausting, so that the residue of gas in it was transparent to the light to be photographed.

In case a vacuum arc was the source, a small and convenient form of water-cooled chamber with removable electrodes was very kindly loaned to me by Professor Paschen (this part of the work having been done in his laboratory at Tübingen). The positive

electrode was itself separately water-cooled, and the negative was movable by means of a screw, for making contact. Aluminium was usually chosen as the material for the negative electrode, mainly because it furnishes good standard lines in this part of the spectrum; the positive (lower) terminal was tipped either with a rod of the metal to be studied, or by a cup-shaped piece in which (for low-melting metals, such as In, Tl, Cd, Pb, etc.) a drop of the molten metal could be placed. The exposure came to an end when all this metal had boiled away. A 60-volt direct-current arc was used; higher voltages made the arc wander away often from the tips of the terminals. An image of the arc was cast on the slit by a quartz lens located in a projecting side tube, which was long enough so that the entire course of the rays outside the grating tube was through gas at the same low pressure as that about the arc itself. The results here presented for Ca and Mg came in part from a more ordinary experimental arrangement, in which the grating and source were used in air. The vacuum-grating arrangement gave results to λ 1670, the sudden increase in absorption due to air at λ 1850 being quite inconspicuous, due to the shortness of the path in air.

The vacuum arc spectra of Mg, Ca, Zn, Cd, Al, In, Tl, Pb, Sn, and Cu were investigated. The plates were sometimes underexposed, as it was difficult to run the arc very long. Cu gave no new results. The "vacuum lamp"[1] of Mg and of Ca was also used, and gave beautifully sharp lines, though not many of them in this region. The spark spectra of Mg, Ca, Zn, Cd, Al, In, and Tl were also obtained. Certain common impurity lines showed themselves in some of these spectra, but are not here mentioned, lacking positive identification. In every case great care was taken to eliminate from the lists lines which could be identified as due to impurities. The wave-lengths chosen as standards were Paschen's measurements of the Zn lines near λ 2100; Runge's of the Al lines near λ 1850, and Wolff's[2] value for Al, λ 1670. In general there is a great difference between the arc in air and the vacuum arc, the latter giving a spectrum nearly the same as that

[1] *Astrophysical Journal*, **40**, 377, 1914.

[2] Wolff, *Annalen der Physik*, **42**, 825, 1913.

from the spark. In Tl and in Ca, for instance, the ends of important series come in this region and are rather prominent in the spectrum of the arc in air, but usually absent from the other spectra altogether.

The results for each element are presented in detail below; the wave-lengths are given in international units, reduced to vacuum. Through most of this part of the spectrum it has been assumed that the wave-lengths on the Rowland scale are too great by 0.06 units.

ALUMINIUM

The vacuum arc spectrum of Al proved to be practically identical with the spark spectrum as given by Lyman. The prominent single line at λ 1670 is perhaps a little stronger relative to the triplet near λ 1720 in the arc than in the spark. This is to be expected if these lines really belong to single-line and triplet series systems, as their appearance would indicate.

MAGNESIUM

In a recent (1913) Tübingen dissertation by E. Lorenser, there appears a single-line series of the "principal" type in the spectrum of Mg, with its first line at λ 2852, the flame line of this element. The second line of this series was not observed by Lorenser, but was calculated by him to be at λ 2026.46 I.A.vac. Some time earlier I had obtained a photograph of this line, which is here reproduced (Fig. 1*a*), showing its heavy reversal, the line thereby declaring itself as a line of the principal series. The arc in air between Mg rods was used as source. The same line shows also in Fig. 1*b*, Ca arc in air, the line not being reversed, since there was not much Mg in the source. Sharp photographs of this line were obtained with vacuum sources, and from these its wave-length has been determined to be λ 2026.48 I.A.vac., differing from Lorenser's calculated value by less than the probable error of measurement. It is worth noting that there is a Zn line whose wave-length is about 0.3 greater, but this is a "pair" line, strong in the spark spectrum and very weak in the arc in air. This is just opposite to the behavior of the Mg line, so that they almost

never occur together. Eder and Valenta give a Cd line here also, but it seems likely that this was due to Zn.

Photographs with the vacuum arc, and with the Mg "vapor lamp" also, show the next line of this same Mg series, for which the value λ 1828.06 is obtained. This agrees with the value given by Lyman, who was the first to measure this line. The spark spectrum of Mg was photographed in this region, but showed no new lines.

CALCIUM

The measurements of the Ca spectrum in the region of λ 2000 given in an earlier paper[1] have since been improved by getting better photographs. One of these is of sufficient interest to be worth reproducing (Fig. 1*b*). It was taken from an arc be-

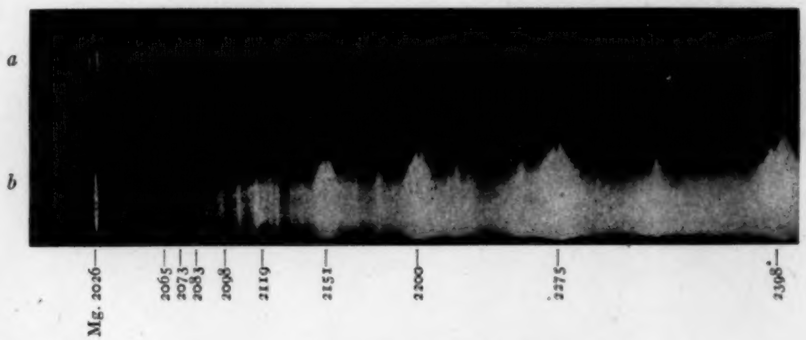

FIG. 1.—Region of λ 2000

a. Mg arc in air, λ 2026 only.
b. Ca arc in air, showing principal series of single lines.

tween two pieces of metallic Ca, of which the positive one was on the point of melting in air. A short-focus concave grating and a Schumann plate were used. The reproduction, enlarged fourfold, shows the reversed lines of the single-line principal series (called SL1 before), heavily overexposed. The emitted light from these lines in the case of the last four fuses together into a continuous background, upon which the reversals stand out as absorption lines. The character of these lines in this photograph is strong evidence for classifying them together as a principal series. The first two lines of this

[1] *Astrophysical Journal*, **32**, 153, 1910.

series are the flame line λ 4226 and a line at λ 2721.76 Rowland (air), which is peculiar in two ways. First, it was discovered by Rowland[1] but was overlooked by later observers, and has not yet found its way into the lists. Second, its intensity is abnormally low as a member of the series. Yet, it must be accepted as one, because its wave-length is precisely that predicted[2] by a simple shift from a line (λ 6718) of another series (SL2), in exactly the same way as are the other lines of this series. This abnormality of intensity is not absolutely unique, as something of the sort occurs in the spectrum of potassium; and it appears that in other sources, such as the "vapor lamp" of Ca, the abnormality is not so striking.

The list below gives the remeasured values of some of the series lines mentioned above, and also several new lines in this part of the spectrum which appear to belong to Ca, and to be members of new single-line series.

λ I.A. Vac.	Intensity	Source	Remarks
2428.77.......	1	Arc in air	New
2392.94.......	2	Arc in air	New
2330.02.......	2	Arc in air	New
2258.07.......	1	Arc in air	New
2222.59.......	1	Arc in air	New
2211.86.......	0	Vapor lamp	New ?Ca
2187.66.......	1	Arc in air	New
2179.44.......	1	Arc in air	New
2167.57.......	0	Arc in air	New ?Sr
2132.51.......	0	Arc in air	New
2098.14.......	1	Arc in air	Prin. series, remeasurement
2083.38.......	1	Arc in air	Prin. series, remeasurement
2073.26.......	0	Arc in air	Prin. series, remeasurement
2065.42.......	0	Arc in air	Prin. series, new
1840.26.......	6	Vapor lamp	Remeasurement
1838.13.......	5	Vapor lamp	Remeasurement

ZINC

The spectrum of Zn obtained from the vacuum arc included only the stronger lines shown in the spark spectrum, in the same relative intensities, possibly on account of underexposure. The spark spectrum was measured in part, with the following results:

[1] *Astronomy and Astrophysics*, **12**, 321, 1893.

[2] *Physical Review*, **1**, 332, 1913.

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2253.55	1	1969.59	1
2139.27*	9	1964.76	1
2104.92	1	1919.04	2
2102.88	3	1839.19	1
2100.53*	9	1833.79	1
2097.44	1	1767.84	2
2087.66	3h	1757.25	1h
2079.57	1	1752.6	0
2064.93	7	1749.59	2
2062.57*	9	1745.73	0
2026.19*	10	1742.98	2
2012.55	1	1706.89†	0
1987.17	1	1689.04	0
1982.21	2	1673.09	1

* Paschen's value, used as standard.

† ?Zn.

The observations of Handke (see Kayser's *Spectroscopy*, Vol. VI) are the only ones extending below λ 1919. The list here given does not agree with his.

CADMIUM

The spectrum of Cd was obtained in the same way as that of Zn, and there was the same similarity between the vacuum arc spectrum and the spark spectrum. The table below of the spark spectrum is in part covered by the measurements of L. and E. Bloch[1] but there are many unexplained differences between the two lists.

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2265.61	9	1856.62	3
2240.53	1	1855.83	1
2195.30	9	1844.58	1
2145.05*	10	1793.06	1
2112.18	2	1789.04	0
2056.80	0	1772.89	2
2027.64	1	1768.54	0
2004.54	2	1747.77	2
1987.84	1	1721.76	0
1903.37	0	1707.04†	0
1874.06	6		

* Used as standard: value of Kayser and Runge.

† ?Cd.

[1] *Journal de Physique*, 4, 622, 1914.

INDIUM

The spark and vacuum arc spectra of indium were found to be as follows:

λ I. A. Vac.	Int. Arc	Int. Spark	λ I. A. Vac.	Int. Arc	Int. Spark
2351.45	....	2	1820.38	....	1
2321.83	....	1	1774.68	....	2
2306.81	5	7	1770.57	1	5
2154.81	....	1	1748.65	1	5
2079.28	4	7	1716.28	....	1
1977.44	2	5	1699.86	....	2
1966.69	2	6	....	....	....

THALLIUM

The spark and vacuum arc spectra of thallium were found to be as follows:

λ I. A. Vac.	Int. Arc	Int. Spark	λ I. A. Vac.	Int. Arc	Int. Spark
2469.76	....	4	2207.58	....	1
2452.70	....	8	2139.98	....	2
2395.27	....	1	1908.68	10	5
2380.34	3	6	1892.72	1	3
2365.65*	....	2	1881.19	....	2
2298.95	4	9	1871.47	....	1
2243.66	....	1	1837.96	....	1
2238.59	....	1	1828.00	....	2
2210.46	....	1	1814.72	1	4

* Not Cd.

LEAD

The vacuum arc spectrum of lead is as follows:

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2402.62	6	2170.60	8
2400.17	1	2115.63	2
2394.52	7	2112.37	1
2389.34	1	2053.83	2
2332.97	2	2022.64	0
2254.68	0	1904.88	1
2247.53	7	1868.59	0
2238.03	3	1822.06	8
2204.18	10	1796.53	3
2190.08	1	1726.71	1
2176.23	0	1682.54	0

The relative intensities in the vacuum arc are not the same as those in the arc in air, but are approximately those of the spark spectrum.

Kayser and Runge found in the spectrum of Pb related groups of lines, one of which is reproduced from another by the addition of a constant frequency-difference. The new lines here given made it worth while to test the spectrum again for such groupings, and some new results were obtained. For instance, the lines λλ 5201, 5005, and 4340 form part of an additional group; as do also, λλ 2833 2170, 2053, 2022, 1904, and 1868. Apparently also λλ 4062 and 2613 belong to Kayser and Runge's first and third groups respectively; λλ 2823 and 2614 to their second and third, and λλ 7228, 4057 and 3639 to their first, second, and third. The careful measurements of Klein[1] were used as a basis for this part of the work.

TIN

Observations similar to those made on Pb were made also with Sn, with the following results (vacuum arc spectrum):

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2355.57	5	2199.93	3
2335.53	2	2195.16*	1
2317.93	2	2152.08	4
2287.28	2	2149.43	1
2269.65	6	2114.74	2
2246.73	4	2092.14	1
2232.37	1	1811.29	3
2210.38	4	....	....

* Not Cd.

In the same manner as with Pb, the groups of lines found by Kayser and Runge in the spectrum of Sn were examined, with similar results. To the first, second, and third groups of Kayser and Runge should be added λ 3662, λ 2790, and λ 2661 respectively. Part of a fourth group is formed by λ 5631, λ 4524, λ 3655, and λ 3141. Several other pairs of unclassified lines have the separation of the first and second groups, or of the second and third, but, as some of these coincidences may be accidental, it may not be worth while to give all the details.

[1] Klein, Bonn Dissertation, 1913.

SUMMARY

The vacuum arc spectra of Mg, Ca, Zn, Cd, Al, In, Tl, Pb, and Sn have been examined from λ 2300 to λ 1670, as have also the spark spectra of Mg, Ca, Zn, Cd, Al, In, and Tl. The results herewith presented are in some cases of interest in their connection with series or other relationships among the lines.

VASSAR COLLEGE
January 1916

Minor Contributions and Notes

THE ALTITUDE OF AURORA BOREALIS SEEN FROM BOSSEKOP DURING THE SPRING OF 1913

In the November 1913 issue of this *Journal* (**38**, 311) I gave a short account of an expedition to Bossekop which I undertook in the spring of 1913 for the purpose of completing the results of my expedition to the same place in 1910.

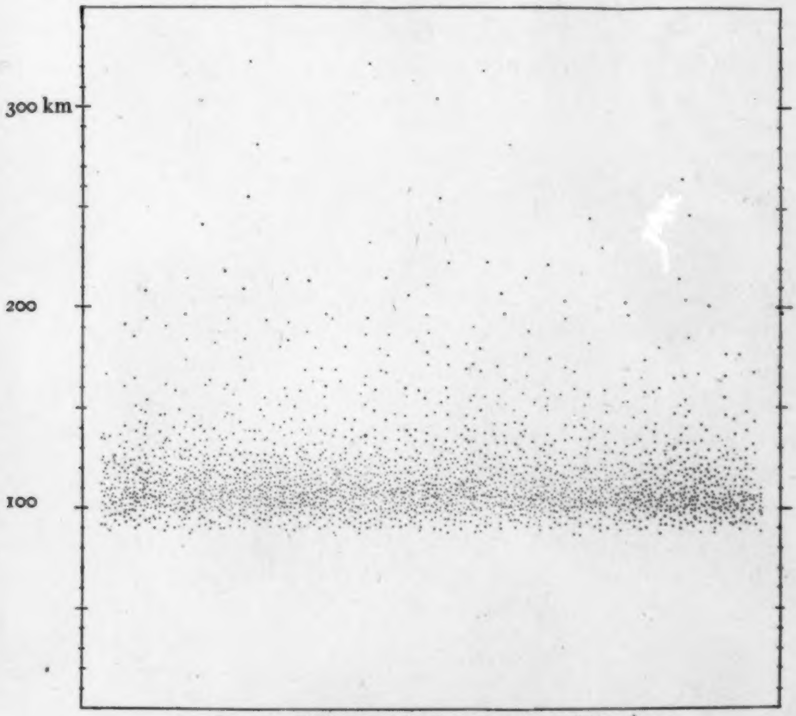

Fig. 1. Altitudes of selected points of Auroras

The aim of the expedition in 1913 was mainly to obtain a greater number of good photographs of the aurora for the determination of its height and situation in space. A very great number of

simultaneous photographs were taken from the two stations Bossekop and Store Korsnes, which are connected by telephone and are situated 27.5 km apart. My assistant, the meteorologist Bernt Johannes Birkeland, made the observations at Store Korsnes and I those at Bossekop.

The material has now been worked up and about 2500 parallaxes of selected points of the auroras have been calculated for the determination of the situation and altitude of these points above the earth's surface. They are all located within a distance not exceeding 800 km from Bossekop and mainly in the directions west, north, and east.

On Fig. 1 are seen all the altitudes calculated, each altitude marked by a point. The number of points giving the same altitude are distributed as follows:

TABLE I

Altitude in Kilometers	Number of Points	Altitude in Kilometers	Number of Points	Altitude in Kilometers	Number of Points	Altitude in Kilometers	Number of Points
85	0	109	75	133	13	157	4
86	1	110	73	134	16	158	3
87	2	111	70	135	12	159	6
88	8	112	72	136	8	160	5
89	11	113	72	137	16	161	2
90	21	114	55	138	10	162	5
91	25	115	67	139	17	163	0
92	28	116	39	140	6	164	4
93	30	117	49	141	8	165	2
94	34	118	54	142	9	166	4
95	36	119	39	143	7	167	2
96	41	120	52	144	3	168	1
97	43	121	24	145	4	169	3
98	48	122	40	146	9	170	2
99	66	123	24	147	9	171	3
100	70	124	28	148	8	172	3
101	84	125	33	149	7	173	1
102	79	126	25	150	0	174	2
103	91	127	27	151	2	175	1
104	72	128	24	152	5	176	1
105	101	129	18	153	3	177	4
106	99	130	15	154	2	178	1
107	95	131	11	155	4	179	0
108	85	132	17	156	4	180	4

Over 180 km there are some altitudes which can be seen on the figure; they are derived particularly from auroral rays.

The numbers here given may perhaps be somewhat modified in details through renewed measuring and computation, but I think the average values are well founded.

We shall not here enter into a detailed discussion of the material, nor into a comparison between the observations and the theory, because these questions are taken up in a series of articles now in process of publication in *Terrestrial Magnetism and Atmospheric Electricity.*

CARL STÖRMER

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February 11, 1916

NOTE ON MR. SHAPLEY'S STUDIES OF THE PERIODS OF ECLIPSING VARIABLES

In the *Astrophysical Journal* for May, 1915 (**41**, 291), and in the *Publications of the Astronomical Society of the Pacific* for June, 1915 (**27**, 135), Mr. Harlow Shapley has made use of his catalogue of eclipsing binary stars in order to investigate whether the two types of periods I have suspected from an inspection of the records of spectroscopic binaries[1] are also indicated in the previous class of variables.

In my opinion it was not to be expected that the eclipsing binaries should give much evidence of the grouping in question. At the time when my article was written I had access to the periods of about 80 eclipsing binaries, the evidence from which (p. 12, *loc. cit.*) was thought by me to be in favor of grouping of the periods.

I had various reasons for being satisfied with that evidence, reasons sufficient to satisfy me also with the very vague signs of the grouping (i.e., the curves of skew-frequency) that have come out of Mr. Shapley's recent statistical studies.

Those reasons are based chiefly on the fact that the eclipsing variables are binaries presented for observation by a process of

[1] S. D. Wicksell, "Contributions to the Statistics of Spectroscopic Binary Stars," *Meddelande från Lunds Observatorium*, No. 63, 1913.

selection that is very much less favorable for the tracing of two groups of periods than is the case with the spectroscopic binaries. Such different modes of selection, within two statistical series that are to be compared, often cause trouble in such investigations, and the mind of the statistician must always be on the alert for the occurrence. In the present instance the mode of selection unduly favors the observation of short-period doubles in case of the eclipsing systems. The selection is produced in the following way: first, the photometric methods are better adapted for the detection and characterization of short-period variables than the spectroscopic methods; second, the probability that a system of double stars will be seen from the earth as an eclipsing system, very rapidly decreases with increasing distance of the components, hence also with increasing period. Also the fact that the systems of small mass are less liable to eclipse than systems of great mass will tend to connect the eclipsing character with shortness of period, for the period is inversely proportional to the square root of the mass.

Hence it is a priori to be expected that a catalogue of eclipsing binaries will contain mainly those of short periods, and that a long-period group after its eventual separation will be indicated only slightly or not at all. Thus the distribution of eclipsing variables will not justify any conclusions regarding the distribution of periods of double stars in general. The whole thing may plainly be illustrated by plotting the curves of *relative* frequency for the logarithm of the period of eclipsing and spectroscopic binaries. The much greater height and steepness (i.e., the smaller dispersion) of the eclipsing curve for short periods is a practical illustration of the statements above.

Finally I will again call attention to a property of the eclipsing binaries first detected by Shapley, which, if not directly indicating two groups of periods, or rather of orbits, may at least be well explained by assuming that they exist. I refer to the tendency of eclipsing variables to show two groups according to density. It is clear that if two groups of orbital dimensions exist, two groups of densities should also appear among eclipsing binaries, while

the eclipsing character of a system of great orbital radius makes it probable that the volumes of the components are also large. Otherwise the components would eclipse only if the inclination were confined within a very narrow range, and the eclipses should be very short, in comparison with the period, and very hard to detect. If the volumes are great, the densities must on the whole be small, as the long-period stars are not generally of very great mass. In the article cited I have already pointed out that Shapley's group of low density emanates from the long-period systems, a fact that I deem very well worthy of notice, as it is in contradiction to the explanation given by Shapley himself for this phenomenon.

On the whole I must say that the results of the statistical studies of Shapley are of a character rather to strengthen the evidence in favor of the existence of two groups of periods among the close double stars. To avoid misinterpretation I must finally assert that processes of selection similar to those here described for the eclipsing binaries are of course also at work on the spectroscopic binaries; but the significant difference lies in the fact that the force of the selection is so much stronger in the former category. This fact is clearly illustrated by the following argument: suppose that by spectroscopic and photometric methods we had discovered and determined the period of all double stars in the heavens down to the seventh magnitude and all shorter than a certain period. Then, taking out all systems that are eclipsing, the above-mentioned second mode of selection should still offset their periods, i.e., eclipses will be relatively less frequent among systems with large orbits (periods) than among systems with small orbits. Thus going back to the present state of the catalogues it is seen that there is always a preponderance in the selection in favor of short periods in case of the eclipsing variables.

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January 2, 1916

ADDENDA AND ERRATA FOR VAN MAANEN'S LIST OF STARS WITH PROPER MOTIONS EXCEEDING $0\overset{''}{.}50$ ANNUALLY[1]

The following numbers of the list in *Mt. Wilson Contr.*, No. 96,[2] (several of which were marked doubtful) should be excluded: Nos. 6, 7, 63, 69, 130, 167, 196, 226, 267,[3] 330, 331, 364, 384, 389, 390, 441, 442, 480, 514, and 526.

Further, in the case of

No. 25 read $\phi = 66\overset{\circ}{.}6$	No. 269 read $\phi = 268\overset{\circ}{.}0$
" 62 " " $= 110.4$	" 325 add β G.C. 7166
" 159 " " $= 206.3$	" 365 read $\phi = 304\overset{\circ}{.}7$
" 162 " " $= 209.3$	" 440 " " $= 11.7$
" 166 " " $= 179.0$	" 499 " " $= 61.9$
" 187 delete Sp. Bin.	" 508 add β G.C. 12274
" 247 read $\phi = 291\overset{\circ}{.}6$	

Attention may be drawn to Porter's note in *Astronomical Journal*, **20**, 46, 1915, which gives several stars whose proper motions, as indicated by later observations, are a little below the limit of half a second.

Besides the additions to the original list which appear in the accompanying table, 12 stars with proper motions in excess of $0\overset{''}{.}50$ have just been given by Wolf in *Astronomische Nachrichten*, **201**, 345, 1915.

[1] This is appended to Mt. Wilson Contr., No. 111.

[2] *Astrophysical Journal*, **41**, 187, 1915.

[3] *Astronomische Nachrichten*, **201**, 271, 1915.

STARS TO BE ADDED TO THE LIST IN *Mt. Wilson Contr.* No. 96; THIS JOURNAL, 41, 187, 1915

Name	B.D.	Mag.	α 1900	δ 1900	μ	φ	Authority
Comp. to Lal. 248	....	10.7*	$0^h12^m43^s$	+43°28′	2″83	82°9	O. Struve
Anonymous	....	13 *	2 8 14	+15 31	1.1−	90.0	Wolf
A. Oe. 2526	+64°312	7.5*	2 9 12	+64 30	0.50	228.5	Po.
Comp. to Pi 2^h123	....	11.5†	2 30 46	+ 6 24	2.63	48.9	Van Maanen
A. Oe. 3303	+51°658	9.1	2 50 18	+52 5	0.50	107.9	Po.
A. Oe. 4961	+52°857	8.7	4 29 47	+52 42	0.53	149.7	Po.
B.D.+17°1320	+17°1320	9.5	6 31 26	+17 38	0.88	293.−	Wolf
B.D.+33°1505	+33°1505	9.3	7 12 58	+33 2	0.57	127.−	Hertzsprung
B.D.+33°1694	+33°1694	9.2	8 18 54	+32 58	0.69	173.−	Hertzsprung
Schroeter 899	+67°552	9.3	8 27 23	+67 38	1.09	270.0	Schroeter
B.D.+77°361	+77°361	9.2	9 6 24	+77 40	1.02	268.0	Jones
Anonymous	....	9.5*	10 56 56	+ 1 19	0.7−	38.−	Wolf
Anonymous	....	13 *	11 2 9	+ 2 9	0.67	146.−	Wolf
Anonymous	....	10.5*	11 4 56	− 2 14	0.49	166.−	Wolf
Anonymous	....	12.5*	11 10 58	+ 8 34	0.81	261.5	Wolf
Anonymous	....	10 *	11 18 39	+ 9 7	1.05	278.7	Wolf
Comp. to Lal. 27744	....	7.5	15 8 50	− 0 58	1.38	247.4	β G.C. 7166
Comp. to W.B. 23^h175	....	8.4	23 11 54	−14 22	1.31	202.9	β G.C. 12274

† Visual magnitude. The photographic magnitude is 13.2. These values were kindly determined by Mr. Shapley.

ADRIAAN VAN MAANEN

MOUNT WILSON SOLAR OBSERVATORY

REVIEWS

Die Kultur der Gegenwart, Dritter Teil, Dritte Abteilung: Physik. Edited by E. WARBURG. Leipzig and Berlin: B. G. Teubner; 1915. 4to. Pp. 762. M. 22; bound, M. 24.

This volume contains thirty-six articles on the principal phases of modern physics, as follows: 1, "Die Mechanik im Rahmen der allgemeinen Physik," by E. Wiechert. 2, "Historische Entwicklung und kulturelle Beziehungen," by Felix Auerbach. 3, "Thermometrie," by E. Warburg. 4, "Kalorimetrie," by L. Holborn. 5, "Entwicklung der Thermodynamik," by F. Henning. 6, "Mechanische und thermische Eigenschaften der Materie in den drei Aggregatzuständen," by L. Holborn. 7, "Umwandlungspunkte, Erscheinungen bei koexistierenden Phasen," by L. Holborn. 8, "Wärmeleitung," by W. Jaeger. 9, "Wärmestrahlung," by Heinrich Rubens. 10, "Theorie der Wärmestrahlung," by W. Wien. 11, "Experimentelle Atomistik," by Ernst Dorn. 12, "Theoretische Atomistik," by A Einstein. 13, "Entwicklung der Elektrizitätslehre bis zum Siege der Faradayschen Anschauungen," by Franz Richarz. 14, "Die Entdeckungen von Maxwell und Hertz," by Ernst Lecher. 15, "Die Maxwellsche Theorie und die Elektronentheorie," by H. A. Lorentz. 16, "Ältere und neuere Theorien des Magnetismus," by R. Gans. 17, "Die Energie degradierenden Vorgänge im elektromagnetischen Feld," by E. Gumlich. 18, "Die drahtlose Telegraphie," by Ferdinand Braun. 19, "Schwingungen gekoppelter Systeme," by Max Wien. 20, "Das elektrische Leitungsvermögen," by H. Starke. 21, "Die Kathodenstrahlen," by W. Kaufmann. 22, "Die positiven Strahlen," by E. Gehrcke, and O. Reichenheim. 23, "Die Röntgenstrahlen," by W. Kaufmann. 24, "Entdeckungsgeschichte und Grundtatsachen der Radioaktivität," by J. Elster and H. Geitel. 25, "Radioaktive Strahlungen und Umwandlungen," by Stefan Meyer and Egon von Schweidler. 26, "Entwicklung der Wellenlehre des Lichtes," by Otto Wiener. 27, "Neuere Fortschritte der geometrischen Optik," by Otto Lummer. 28, "Spektralanalyse," by F. Exner. 29, "Struktur der Spektrallinien," by E. Gehrcke. 30, "Magnetooptik," P. Zeeman. 31, "Verhältnis der Präzisionsmessungen zu den allgemeinen Zielen der Physik," by E. Warburg. 32, "Die Erhaltung der Energie und die Vermehrung der

Entropie," by F. Hasenöhrl. 33, "Das Prinzip der kleinsten Wirkung," by Max Planck. 34, "Die Relativitätstheorie," by Albert Einstein. 35, "Phänomenologische und atomistische Betrachtungsweise," by W. Voigt. 36, "Verhältnis der Theorien zueinander," by M. Planck.

Each article is a very brief but thoroughly correct and modern statement of the subject which it discusses. The book is intended primarily for the general academic public. The physicist who expects to find here a great store of modern data in any particular field will be disappointed, as the articles are necessarily very brief and are written in popular style, as free as possible from technicalities and practically without equations. They may be read with profit, however, by physicists who desire a brief, scholarly summary. The strongest appeal of the book should be to workers in allied sciences—chemistry, astronomy, technology, biology, etc.—who have been unable to keep up with the rapid development of physics in recent years.

H. G. G.

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